Aspects and Motions of the Moon: Eclipses

The moon, the earth's nearest celestial neighbor and its only known natural satellite, accompanies the earth in its annual revolution about the sun. Although the moon shines only by reflected sunlight, it is nevertheless the second most brilliant object in the sky, and is one of the three celestial objects (the sun, the moon, and Venus) that can be seen in broad daylight. Many of the ancients regarded the moon as "ruler of the night" just as the sun is "ruler of the day." It was one of the first objects to be studied with the telescope and is the celestial body about which we know the most.

Having examined, in the last two chapters, the motions of the earth and how those motions affect the appearance of the sky, we now turn our attention to the apparent and real motions of the moon and certain other of its aspects. The physical nature of the moon itself, however, is discussed in Chapter 13.

9.1 ASPECTS OF THE MOON

The moon, because of its proximity, appears to move more rapidly in the sky than any other natural astronomical object, except meteors, which are within the earth's atmosphere. The moon appears the same size as the sun in the sky, subtending ½° of arc. Its larger surface features are easily visible to the unaided eye and form the facial markings of the "man in the moon." As it travels about the earth each month, it displays different parts of its daylight hemisphere to our view and progresses through its cycle of phases.

(a) Moonlight

The most conspicuous property of the moon is its light. The amount of light we receive from the moon varies immensely with its phase. When the moon is full, its light is nearly bright enough to read by; we receive only about 10 percent as much

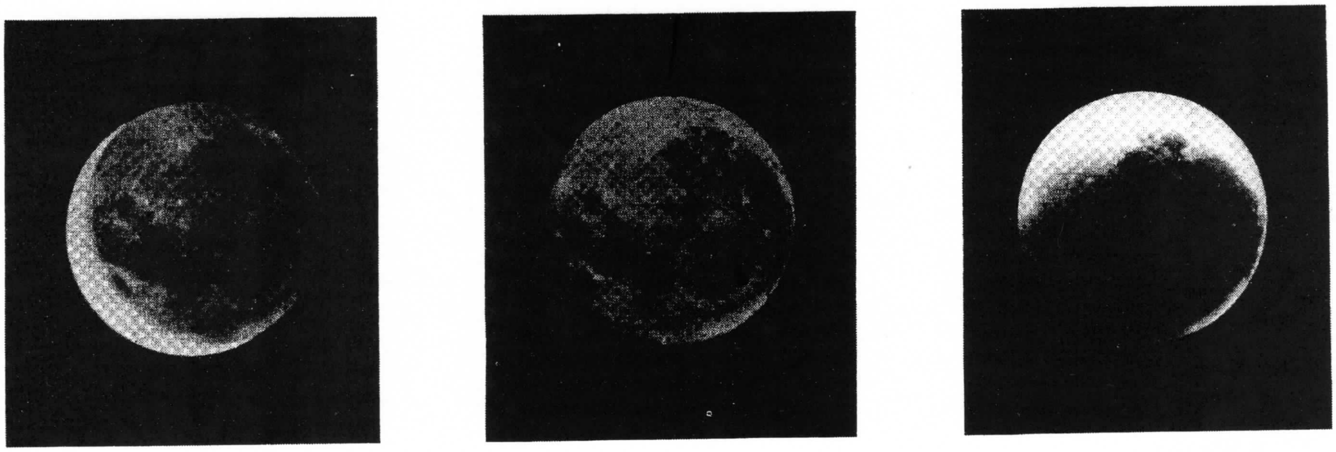


FIG. 9-1 The earth-lit, the full, and the totally eclipsed moon. *(Yerkes Observatory.)*

light from the moon at first and last quarter, and only one thousandth of the light of the full moon when the moon appears as a thin crescent 20° from the sun in the sky.

Despite the brilliance of the full moon, it shines with less than 1/400,000 the light of the sun. Even if the entire visible hemisphere of the sky were packed with full moons, the illumination would be only about one fifth or less of that in bright sunlight.

Because the moon shines by reflected sunlight, we can calculate the moon's reflecting power from its apparent brightness. The moon and earth are at about the same distance from the sun; consequently, the moon receives as much sunlight per square inch of its surface as does the earth. Calculation shows that if all this light were reflected back into space, the full moon would appear about 14 times as bright as it actually is. The fraction of incident light that is reflected by a body is called its *albedo*. The average albedo of the moon is thus about 0.07. The moon absorbs most of the sunlight that falls upon it; its surface is quite dull. The absorbed energy heats up the surface of the moon until the energy is radiated away again as radiant heat.

When the bright part of the moon appears as only a thin crescent, the "night" side of the moon often appears faintly illuminated. Leonardo da Vinci (1452–1519) first explained this illumination as *earthshine*, light reflected by the earth back to the night side of the moon, just as moonlight often illuminates the night side of the earth.

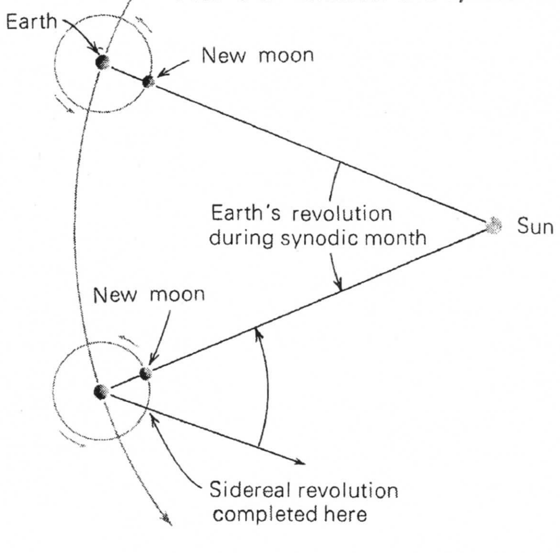
(b) Sidereal and Synodic Months

The moon's sidereal period of revolution about the earth, that is, the period of its revolution with respect to the stars, is $27^{d}7^{h}43^{m}11\overset{s}{.}5$ (27.32166 days). However, during this period of the moon's sidereal revolution, the earth and moon together revolve about $\frac{1}{13}$ the way around the sun, or about 27°. The sun, therefore, appears to move 27° to the east on the celestial sphere during the period of the moon's sidereal revolution. In other words, the moon would not, in its sidereal period, have completed a revolution about the sky with respect to the sun, and consequently would not have completed a cycle of phases. To complete a revolution with respect to the sun, the moon requires, on the average, $29^{d}12^{h}44^{m}2\overset{s}{.}8$ (29.53059 days). We have, then, two kinds of month: the *sidereal month*, the period of revolution of the moon with respect to the stars, and the *synodic month*, the period with respect to the sun (Figure 9-2). (Compare with the concept of sidereal and solar days, Chapter 8.)

(c) The Moon's Apparent Path in the Sky

If the moon's position among the stars on the celestial sphere is carefully noted night after night, it is seen that the moon changes its position rather rapidly, moving, on the average, about 13° to the east per day. In fact, during a single evening the moon can be seen visibly creeping eastward among the stars. The moon's apparent path around the celestial sphere is a great circle (or very nearly so) that intersects the sun's path, the ecliptic, at an angle of about 5°.

FIG. 9-2 Sidereal and synodic months.



The moon's path intersects the ecliptic at two points on opposite sides of the celestial sphere. These points are called the *nodes* of the moon's orbit. The node at which the moon crosses the ecliptic while moving northward is called the *ascending node,* and the node at which the moon crosses the ecliptic moving southward is the *descending node.*

The moon's orbit is constantly and gradually changing because of perturbations, just as the orbits of artificial satellites change. The most important of the perturbations are caused by the gravitational attraction of the sun. One of the effects of the perturbations on the moon's orbit is that the nodes slide westward along the ecliptic, completing one trip around the celestial sphere in about 18.6 years. This motion is called the moon's *regression of the nodes.* Perturbations also cause the inclination of the moon's orbit to the ecliptic to vary from 4°57′ to 5°20′; the average inclination is 5°9′.

If it were not for the regression of the nodes, the moon's orbit would maintain a nearly fixed angle to the celestial equator. It maintains a nearly fixed angle of about 5° to the ecliptic, but because the nodes constantly shift, its angle of inclination to the equator varies from 23½ + 5°, or about 28½°, to 23½ − 5°, or about 18½° (23½° is the inclination of the ecliptic to the celestial equator).

(d) Delay in Moonrise from Day to Day

We have seen that the moon's average eastward motion with respect to the stars is about 13° per day. The sun, on the other hand, apparently moves to the east about 1° per day. With respect to the sun, therefore, the moon moves eastward about 12° per day. As the earth turns on its axis, the moon, like other celestial objects, appears to rise in the east, move across the sky, and set in the west. But because of its daily eastward motion on the celestial sphere, it crosses the local meridian each day about 50 minutes later, on the average, than on the previous day. We could define this interval of 24^h50^m (see also Section 8.1) as the average length of an apparent lunar day.

Conditions similar to those that cause the length of an apparent solar day to vary also cause apparent lunar days to vary in length. The moon's true orbit is an ellipse; the moon's orbital speed consequently varies, in accordance with Kepler's law of areas. The moon's eastward progress in its orbit is therefore not constant. Moreover, since the moon's orbit is inclined to the celestial equator, the moon's *eastward* progress in the celestial sphere (that is, the projection of the lunar motion on the celestial equator) would not always be uniform even if the moon did move uniformly in its orbit. The daily retardation in successive transits of the moon across the local meridian ranges from 38 to 66 minutes.

Moonrise (and moonset) is similarly retarded from day to day. If the moon did not move with respect to the sun, it would rise at nearly the same time from one day to the next. At moonrise, the moon occupies some particular place on the celestial sphere. Approximately 24 hours later, the same place on the celestial sphere rises again, but the moon in the meantime has moved off to the east, so moonrise does not occur until a little later. At the equator, the daily delay is the same as the moon's delay in crossing the meridian. However, at other latitudes the moon and stars rise obliquely to the horizon, rather than in a direction perpendicular to it. Consequently, the time required for the earth to turn the sky westward through the angle representing the moon's eastward motion is not necessarily the same as the daily delay in moonrise. The phenomenon of the harvest moon, discussed below, provides an excellent example. In the northern parts of the United States, the daily delay in moonrise can vary from a few minutes to well over 1 hour.

(e) The "Harvest Moon"

The *harvest moon* is the full moon that occurs nearest the autumnal equinox. Because the moon, when full, is opposite the sun in the sky, it must rise as the sun sets. When the sun is near the autumnal equinox, the full moon is near the vernal equinox, so at the time of the harvest moon the vernal equinox is rising with the full moon. When the vernal equinox is rising, the ecliptic makes its minimum angle with the horizon for an observer in intermediate northern latitudes.

Since the moon's orbit lies within 5° of the ecliptic, it is evident that at the same hour on successive nights the moon's apparent motion, being nearly parallel to the horizon, will not change the moon's relation to the eastern horizon appreciably. In Figure 9-3, the full moon (position 1) is shown rising at sunset. At the same time on the next night it has moved about 12° along its orbit; but it is not very far below the horizon and will rise by moving along the line *AB*, parallel to the celestial equator. The earth will not have to turn far to bring up the moon on this second night. Thus, for several nights near full moon in late September or early October there will be bright moonlight in the early evening – a traditional aid to harvesters. The phenomenon of the harvest moon is most striking in far northern latitudes.

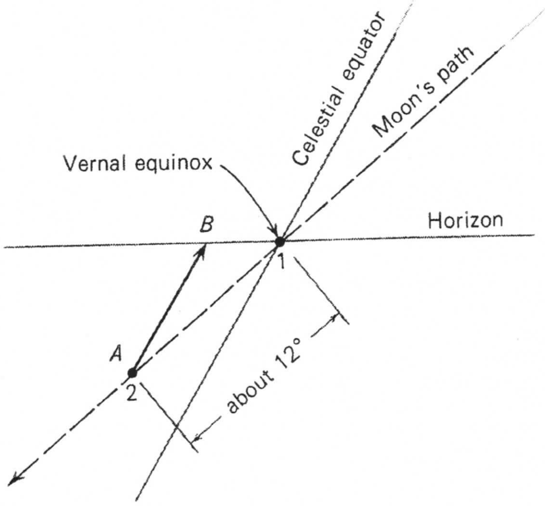


FIG. 9-3 The harvest moon.

(f) The Progression of the Moon's Phases

The phases of the moon (explained in Section 2.2d) were thoroughly understood by the ancients. The relation between any phase of the moon and the moon's corresponding position in the sky at any time of day should now be clear. Except at extreme northern or southern latitudes, one can easily tell where to look for the moon in the sky, if he knows its phase, from a consideration of Figure 9-4.

In Figure 9-4 we imagine ourselves looking down upon the earth and the moon's orbit from the north. The moon is shown in several positions in its monthly circuit of the earth: 1, 2, 3, and so on. The sun is off to the right of the figure at a distance so great that its rays approach the earth and all parts of the moon's orbit along essentially parallel paths. The daylight sides of the earth and moon – the sides of those bodies turned toward the sun – are indicated. For each position of the moon, its phase, that is, its appearance *as viewed from the earth*, is shown just outside its orbit. Several observers are at various places on the earth, *A*, *B*, *C*, and so on. The time of day, indicated for each observer, depends on the position of the sun in the sky with respect to his local meridian or, equivalently, on his position on the earth with respect to the meridian, where it is noon.

For person *A* it is 3:00 P.M. If he sees the moon on his meridian, it must be in the waxing crescent phase (the moon can be seen easily at noon when in this phase). If the moon is in the waning crescent phase it is setting, for it lies on his western horizon. West is the direction away from which the turning earth carries the observer, and his horizon lies in a plane tangent to the surface of the earth at the point where he is standing. If the moon is new it is in about the same direction as the sun in the western sky, and if it is at first quarter it is in the eastern sky. If the moon is rising it must be in the waxing gibbous phase.

For person *B* it is 6:00 P.M. (Person *B* could be person *A* 3 hours later. During a period of even

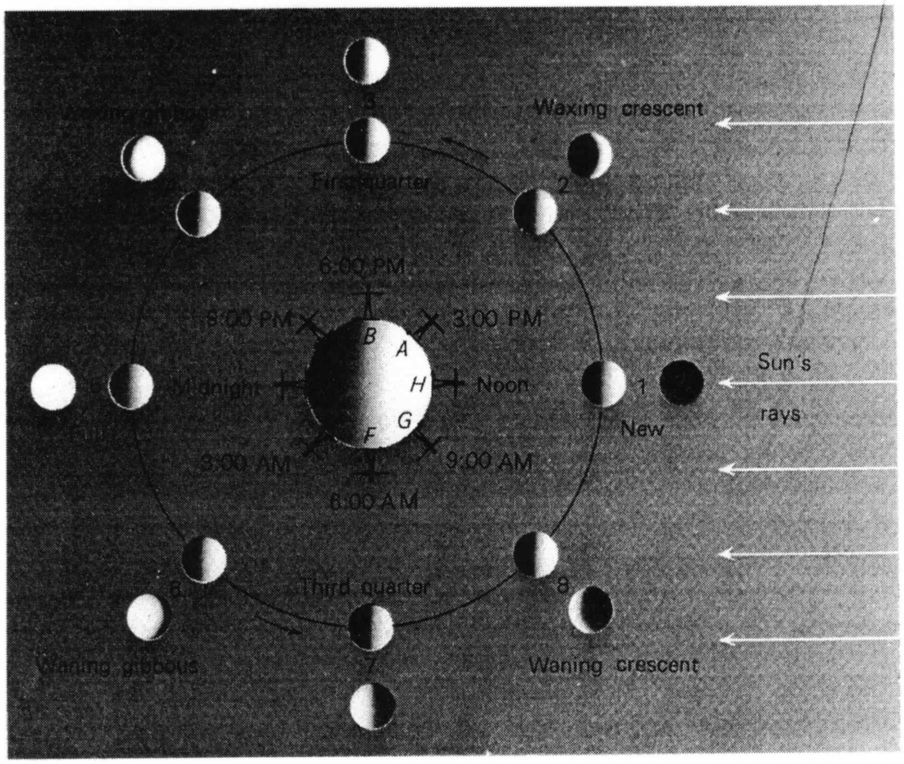


FIG. 9-4 Phases of the moon and the time of day. (The outer series of figures shows the moon at various phases as seen in the sky from the earth's surface.)

a full day the moon does not move enough for its phase to change appreciably.) For *B* the moon is setting if new, and rising if full. If it is a waxing crescent or gibbous it is in the western or eastern sky, respectively. If it is in the first quarter phase, it appears on the meridian.

For person *D* it is midnight. If the moon is full, it rose at sunset, and is now on the meridian. The first or third quarter moon is just setting or rising, respectively. The waxing or waning gibbous moons must appear in the western or eastern sky.

By studying Figure 9-4 we can see that as the moon goes about the earth during the month and gradually changes its phase, its approximate position in the sky is completely determined for any time of day or night. For example, the full moon rises at sunset and sets at sunrise. The first quarter moon rises at noon and sets at midnight, and so on.

Let us take an example. Suppose the waning crescent moon, 30° west of the sun, is observed to rise. We desire to find the time of day. First, we draw a picture of the earth and indicate the direction of the sun and the earth's daylight side (Figure 9-5). Next we draw in the moon 30° west of the sun. To see the moon rising, it must be in the direction of the eastern horizon. (We have exaggerated the size of the earth in Figures 9-4 and 9-5. Actually the direction along the horizon to the moon is very nearly parallel to the line from the center of the earth to the moon.) The only place where that is possible is for the observer shown, where the time is about 4:00 A.M.†

In the foregoing discussion we have ignored the dependence of the moon's position on the latitude of the observer. However, for the most places

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†Suppose your best beau told you how beautiful the moon was as it rose the other "night," and that you know the moon was a waning crescent that couldn't have risen until about 4:00 A.M. If you learn something of the habits of the moon, you may learn something of the habits of your best beau!

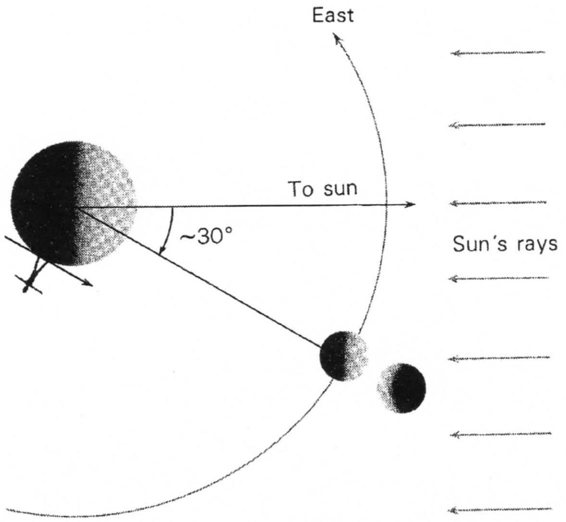


FIG. 9-5 The waning crescent moon when it is rising.

on earth the scheme outlined illustrates well enough where one should look for the moon in the sky.

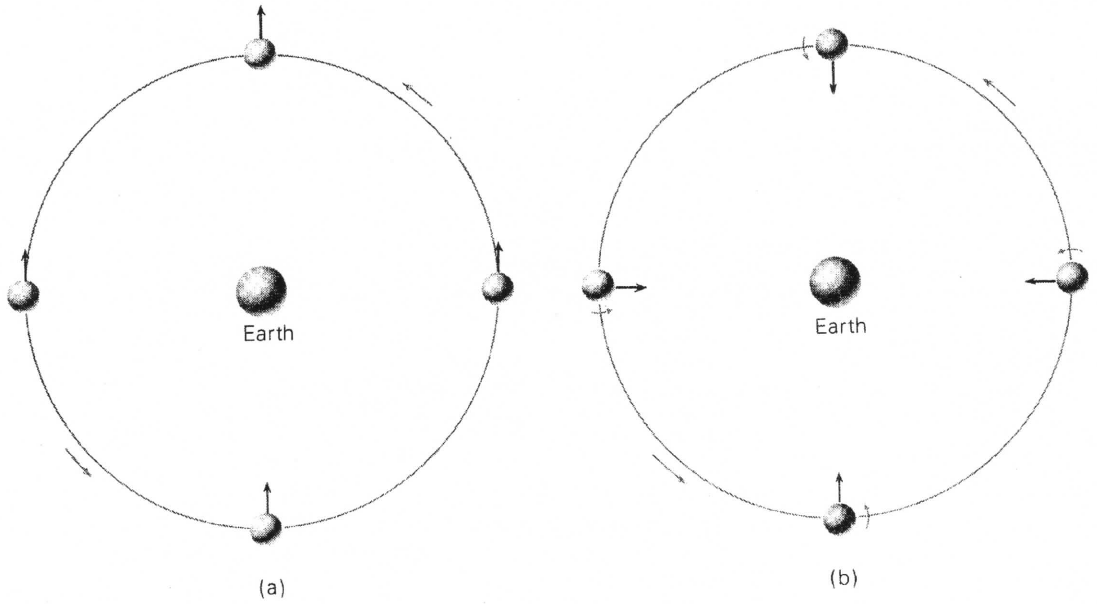
(g) Configurations of the Moon

The configurations of the moon are named like those of the planets. The first or third quarter moon is at *quadrature*. The full moon is at *opposition*, and the new moon is at *conjunction*.†

(h) The Rotation of the Moon

Even naked-eye observation is sufficient to determine that as the moon goes about the earth it keeps the same side toward the earth. The same facial characteristics of the "man in the moon" are always turned to our view. Because the moon always presents the same side to us, it is sometimes said not to rotate on its axis. However, this statement is incorrect. In Figure 9-6 the arrow on the moon represents some lunar feature. If the moon did not rotate, as in (a), we would see that feature part of the time, and part of the time we would see the other side of the moon. Actually the moon rotates on its axis with respect to the stars in the same period as it revolves about the earth, and so always turns the same side toward us (b).

FIG. 9-6 If the moon did not rotate, (a) it would turn all its sides to our view; actually it does rotate, (b) in the same period as it revolves, so we always see the same side.



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†Either full or new moon is also called *syzygy*. It is a term seldom used by astronomers, but it is often encountered in crossword puzzles.

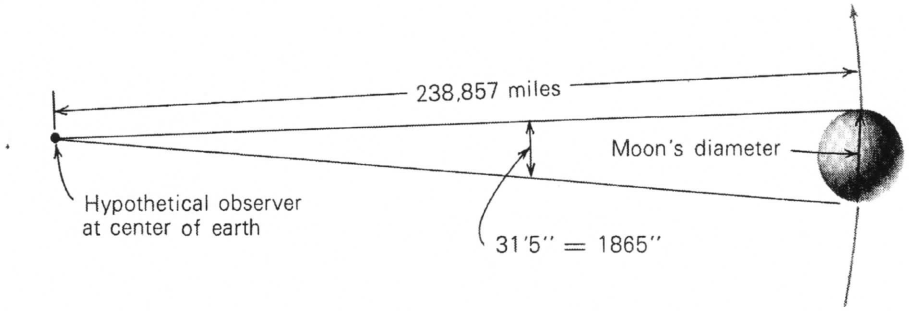


FIG. 9-7 Measuring the moon's diameter.

(a) Radar Measures of the Moon's Distance

The technique of radar is that of transmitting radio waves to a distant object and then detecting those waves that are reflected by that body back to the original source. The direction from which the reflected waves come indicates the direction of the body that reflects them. The time the waves spend making the round trip (the speed of transmission of the radiation being known) indicates the distance of that object.

We know the moon's direction from optical observations, and so are interested only in the time required for the radar signals to travel to the moon and back. The radar signals, being radio waves (a form of electromagnetic radiation), travel with the speed of light – an accurately known quantity. The interval of time between the instant that a wave is broadcast to the moon and the instant it returns can be measured electronically to within a few millionths of a second. The distance to the moon is, of course, the speed of the waves multiplied by half the time required for the round trip.

The first radar contact with the moon was achieved by the United States Army Signal Corps in 1946. A very accurate determination of the moon's distance by this technique was made by the Naval Research Laboratory in 1957. The latter measure gives for the mean distance from the center of the earth to the center of the moon the value 384,403 km, or 238,857 mi. The NRL scientists consider their measure correct to within about 1 mi.

(b) The Moon's Diameter

The mean angular diameter of the moon is 31′ 5″. From its angular size and its distance, the linear size of the moon can easily be found. Because the method is the same as that applied to measure the diameters of planets and other astronomical objects that subtend a measurable angle, we shall explain the procedure in detail.

Notice in Figure 9-7 that because the moon's angular size is relatively small, its linear diameter is essentially a small arc of a circle, with the observer as center and with a radius equal to the moon's distance. Obviously, the moon's diameter is the same fraction of a complete circle as the angle subtended by the moon is of 360°. A complete circle contains 1,296,000″ (there are 60″ per minute, 60′ per degree, and 360° in a circle). As seen from the center of the earth, the moon's mean angular diameter of 31′5″, or 1865″, is thus $1/695$ of a circle. The moon's diameter, therefore, is $1/695$ of the circumference of a circle of radius 238,857 mi. Since the circumference of a circle is 2π times its radius, we have

$$\text{diameter of moon} = \frac{2\pi(238{,}857)}{695} = 2160 \text{ mi.}$$

This type of calculation can be generalized if we note that the distance along the arc of a circle of radius R subtended by 1″ is $2\pi R/1{,}296{,}000 = R/206{,}265$. Thus, if an object, say a planet, subtends an angle of α seconds, and has a distance D, its linear diameter d is given by

$$d = \frac{\alpha D}{206{,}265}.$$

Accurate calculations of the moon's diameter give as a result 3475.9 km, or 2159.86 mi, with an uncertainty of a few hundredths of a mile. This is the moon's equatorial diameter in a direction perpendicular to its direction from the earth. We have already seen (Section 9.1i) that the diameter differs slightly in different directions. The moon's diameter is a little over a quarter that of the earth's, and its volume is about $\frac{1}{49}$ of the earth's.

In practice, of course, we do not observe the moon from the center of the earth but from its surface. Both the angular size and direction of the moon differ for different observers on the earth. However, because the moon's distance and the size of the earth are known, the measured angular size of the moon can easily be corrected to the value it would have if the moon were viewed from the earth's center. The standard reference tables, such as the *Astronomical Ephemeris,* tabulate the moon's position in the sky, its distance, and its angular diameter as seen from the earth's center. To use these tables the astronomer or navigator must correct the tabulated values to those appropriate to his nongeocentric position. To aid in these corrections, the tables also give the moon's *horizontal parallax,* the angle at the moon subtended by the equatorial radius of the earth.

9.3 THE TRUE ORBIT OF THE MOON

When the moon's distance is measured during different times of the month, it is found to vary by more than 10 percent. In obedience to Kepler's first law, the moon's orbit, basically, is an ellipse with the earth at one focus. The sun, however, produces strong perturbations on the moon, and the elements of its orbit can be stated only in an average sense. Indeed, the moon's orbit changes so rapidly that if the moon's positions over a month, relative to the center of the earth, are plotted carefully, even on a sheet of standard typing paper, the orbit is seen to not close on itself. The celestial mechanics of the moon's motion (lunar theory) is very complex; here we can only mention briefly a few of these complexities.

It is easy to visualize one of the effects of the sun's perturbations. The sun's tidal force on the earth-moon system tends, on the average, to separate the two. At new moon, the sun pulls on the moon more strongly than on the earth, tending to pull them apart, and at full moon the sun's greater pull on the earth has the same result. At first and third quarters the sun tends to converge the earth and moon together slightly (because the sun is in slightly different directions as seen from the earth and moon), but this effect is weaker than the tendency to separate the two at full and new moon. Consequently the month is about 53 minutes longer than it would be in the sun's absence.

The tidal force of the sun also acts to speed up and slow down the moon in different parts of its orbit. The net result of these accelerations is to cause the *line of apsides* (the major axis of the moon's orbit) to rotate toward the east in the orbital plane in a period of 8.85 years. Thus the position in the sky of the moon's *perigee* (closest approach of the moon to the earth) advances to the east about the sky in this period.

The plane of the moon's orbit is inclined at about 5°8′ to the ecliptic plane, and one component of the sun's force on the moon is toward the ecliptic plane, thus trying to pull the moon into it. Analogous to precession, however, the moon's orbit does not tilt back into the plane of the ecliptic, but rather the line of nodes regresses (that is, the nodes slide westward), moving around the ecliptic in 18.6 years.

The solar perturbations on the moon's motion depend critically on the average distance of the sun. Because of planetary perturbations on the earth the eccentricity of the earth's orbit slowly oscillates in value. In the present era the eccentricity is decreasing, resulting in a slight and slow increase in the mean distance of the sun. One effect is to shorten slightly the length of the month. The effect was discovered by Edmund Halley in the course of his investigation of ancient eclipse records. The explanation was later supplied by Laplace.

Because of the above (and other) perturbations on the moon, the eccentricity of its orbit changes even during 1 month. The mean value, over many years, is 0.0549. The moon, however, may come as close as 221,463 mi from the earth's center and pass as far from it as 252,710 mi. At its farthest, however, the moon is still only about $\frac{1}{370}$ as far as the sun, and its orbital speed of about $\frac{3}{5}$ mi/sec is only about $\frac{1}{30}$ of the earth's orbital speed about the sun. Thus, if the moon's actual path with respect to the sun is plotted, it is seen to vary only minutely from the earth's orbit; in fact, the moon's orbit is always concave to the sun.

The rapid changes in the moon's orbit make it useful to define two additional kinds of months, which are especially useful in the prediction of eclipses. They are the *nodical* or *draconic month* of 27.2122 days, the time required for the moon to make two successive passages of the same node, and the *anomalistic month* of 27.555 days, the time between two successive perigee passages.

(a) Mutual Revolution of the Earth-Moon System

We saw in Sections 4.4c and 4.4e that one body does not strictly revolve about another, but that the two bodies mutually revolve about their center of mass, or the *barycenter*. It is the barycenter of the earth-moon system that revolves annually in an elliptical orbit about the sun, while the earth and moon simultaneously revolve about the barycenter in a shorter period – the sidereal month.

The elliptical orbit of the center of the earth about the barycenter constitutes an independent motion of the earth. The motion can be detected by careful observations of the nearer planets, or better yet, of near-approaching minor planets (Chapter 15). The motion of Mars, for example, shows monthly oscillations, carrying it a little ahead and then a little behind its regular orbital motion. This oscillation is only apparent and results from the motion of the earth, carrying us first to one side and then to the other side of the barycenter. When Mars is at its closest, the apparent displacements caused by the orbital motion of the earth around the barycenter amount to about 17″. The corresponding mean distance of the center of the earth from the barycenter is 2903 mi. Thus, the earth and moon jointly revolve about a point approximately 1000 mi below the surface of the earth.

(b) The Mass of the Moon

In Section 5.1 we saw that the distances of two bodies from their barycenter are inversely proportional to their masses. The 2903-mi distance of the earth from the barycenter is 1/82.3 of the distance from the earth to the moon; hence the moon is 81.3 times as far from the barycenter as the earth and so is only 1/81.3 as massive. Because the earth's mass is 6.6×10^{21} tons (Section 4.3e), the moon's mass is $(6.6 \times 10^{21})/81.3 = 8.1 \times 10^{19}$ English tons (7.35×10^{19} metric tons).

The best modern determinations of the moon's mass are obtained by measuring the accelerations the moon produces on space probes – either those sent to the moon itself, or that pass it by on interplanetary missions. Analysis of the motion of the Mariner V probe, which passed about 6000 mi from Venus on October 19, 1967, gives for the earth-moon mass ratio the value 81.3004 ± 0.0007; this is the best determination at the time of writing.

(c) *Friction in the Tides and Tidal Evolution**

One force that is believed to affect, over long times, the mutual revolution of the earth-moon system is friction in the tides. As the earth rotates, the tidal waters are continually flowing back and forth over each other, across ocean floors, and in and out over coastal shallows. The resulting friction within the water and between the water and the solid earth draws a considerable amount of energy from the kinetic energy of rotation of the earth and expends this energy in the form of heat. Even friction in the tidal distortions of the solid earth and in atmospheric tides may play a role. The earth, consequently, is slowing in its rate of rotation, and the day is gradually lengthening at the rate of about 0.0016 second per century. The continuous dissipation of the rotational energy of the earth is calculated to be approximately 2 billion horsepower.

The slow increase in the length of the day is very small compared to the short-period fluctuations in the earth's rotation that were described in Section 8.1k; however, those rapid changes in the earth's rotation are *periodic*, which means that at the end of each cycle of changes the earth returns to its original rotation rate. The slow increase in the length of the day due to tidal friction, on the other hand, is a *secular* change, which means that the day continues to lengthen as time goes on, and the changes accumulate until they become noticeable. Suppose a clock keeping accurate mean solar time had been started 2000 years ago. Today that clock would be out of step with the position of the sun in the sky by about 3 hours. This "clock error" was discovered by comparing the locations on earth where ancient eclipses actually occurred and where they would be predicted to have occurred if the earth were not slowing. Those locations depend on the exact times the eclipses occurred. Modern eclipses, predicted from the times of ancient ones, consistently take place too early. A slowing of the rotation of the earth, and a consequent increase in our unit of time, is the only explanation consistent with gravitational theory.

It can be shown that whereas the earth slows down in its axial spin as it loses kinetic energy, the angular momentum of the earth-moon system must be conserved. According to a theory of tidal evolution worked out by Sir George Darwin (son of the naturalist), the moon must slowly spiral outward away from the earth, thus maintaining the constancy of the total angular momentum of the earth and moon. (But this is superimposed on a temporary *decrease* of the moon's distance caused by a lowering of the eccentricity of the earth's orbit.) As the moon's distance from the earth increases, its period of revolution must also increase and its orbital speed must decrease, in accord with Kepler's third law. The day and the month, in other words, will both lengthen. Eventually the day will catch up with the month in length, when both require about 47 of our present days.

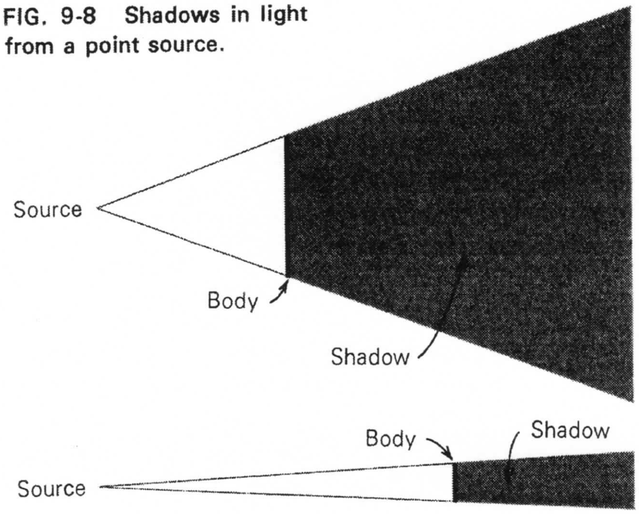
With the day and month of equal duration, the earth and moon will present the same faces to each other. The earth, therefore, will no longer rotate under the tidal bulge produced by the moon, and friction due to lunar tides will have ceased. The solar tides, however, will still be at work. The small friction due to these remaining solar tides on the earth will cause the day to lengthen still more, so that it will then be *longer* than the month. The friction of the lunar tides will consequently return and this time will tend to accelerate the earth's rotation rather than retard it. The friction of solar and lunar tides, in other words, will work against each other. According to the theoretical predictions, the angular momentum associated with the mutual revolution and rotation within the earth-moon system can then be gradually transferred to that associated with the orbital motion of the system as a whole about the sun. As a result, the moon should spiral back toward the earth again.

It has been suggested that eventually, if this course of evolution should proceed to its ultimate end, the strong tidal forces of the earth on the moon would tear the moon apart, leaving a ring of particles revolving about the earth, like the rings of Saturn. It must be emphasized that these changes are exceedingly slow, and even if the predictions and assumptions on which they are based are correct, the entire cycle involves billions of years. It is doubtful, to say the least, that the earth and moon will remain in their present forms long enough for such a course of evolution to take place.

9.4 SHADOWS AND ECLIPSES

Important phenomena associated with the motions of the earth-moon system are eclipses. Eclipses occur whenever any part of either the earth or the moon enters the shadow of the other. When the moon's shadow strikes the earth, people on earth within that shadow see the sun covered at least partially by the moon; that is, they witness a *solar*

FIG. 9-8 Shadows in light from a point source.



eclipse. When the moon passes into the shadow of the earth, people on the night side of the earth see the moon darken – a *lunar eclipse.*

A shadow is a region of space within which rays from a source of light are obstructed by an opaque body. Ordinarily, a shadow is not visible. Only when some opaque material, which will show the contrast between lighted and unlighted areas, intersects the shadow and the surrounding area does the shadow become visible.

(a) Shadow from a Point Source

If a source of illumination is a point source, the shadow cast by an opaque body has sharp boundaries. From a point inside the shadow, the light source is not visible; outside the shadow it is. The boundaries of the shadow diverge radially from the source. The more distant the source, the smaller is the angle of divergence. If the source is infinitely distant, the boundaries of the shadow are parallel (see Figure 9-8).

(b) Shadow from an Extended Source

If a light source is not a point source but presents a finite angular size as seen from the opaque body, as is almost always the case, the shadow cast by the body is not limited to the inner part of the shadow, the *umbra,* where complete darkness prevails. From within the umbra, of course, no part of the light source is visible. At any point completely outside the shadow, there is no obscuration of light, and from such places the entire light source is visible. Between the umbra and the region of full light lies a space of partial illumination, within which the illumination ranges from complete darkness at the boundary of the umbra to full illumination at the outer boundary of the entire shadow. This transition region of the shadow is called the *penumbra.* From any point within it, a part, but not all, of the light source is visible.

As an illustration, consider the shadow cast by a spherical body, such as the earth, moon, or a planet, in sunlight (Figure 9-9). It is obvious from

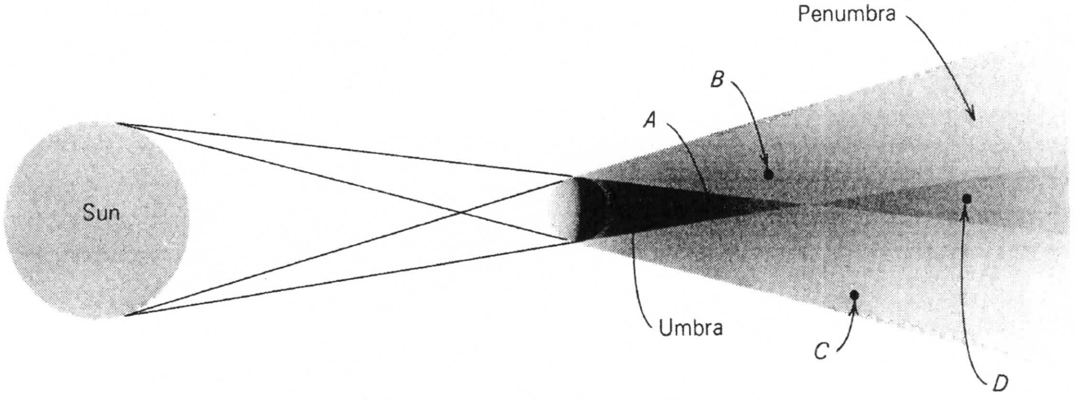


FIG. 9-9 Shadow cast by an opaque spherical body in sunlight.

the figure that because the umbra includes that region of space from which no part of the sun is visible, it must have the shape of a cone, pointing away from the sun. The umbra of the moon or a planet is sometimes called the *shadow cone* of that body. Everybody on the night side of the earth is within the umbra, or shadow cone, of the earth.

The penumbra, on the other hand, is that region from any point within which only part of the sun is covered by the eclipsing body. It is clear from the figure that the penumbra has the shape of a truncated cone pointed *toward* the sun, and that it includes the umbra, as a reversed cone symmetrical about the same axis. The appearance of the sun, as seen from points *A*, *B*, *C*, and *D* in the shadow, is shown in Figure 9-10.

The size of the umbra cast by an opaque sphere in sunlight depends on the size of the sphere and its distance from the sun. The length of the umbra is directly proportional to the distance of the sphere from the sun. Figure 9-11 illustrates this relation.

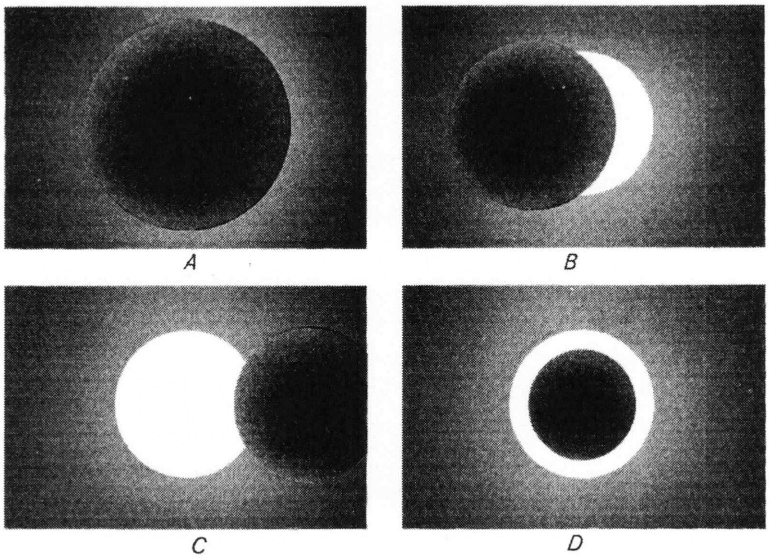


FIG. 9-10 Appearance of the sun from various parts of a shadow.

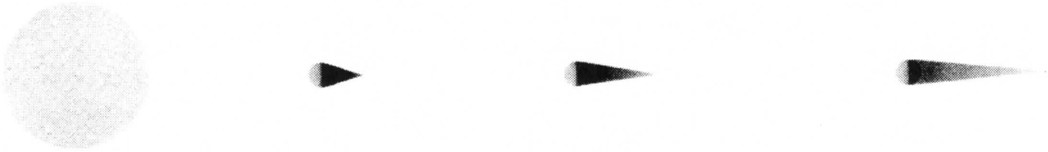


FIG. 9-11 Shadow lengths at various distances from the sun.

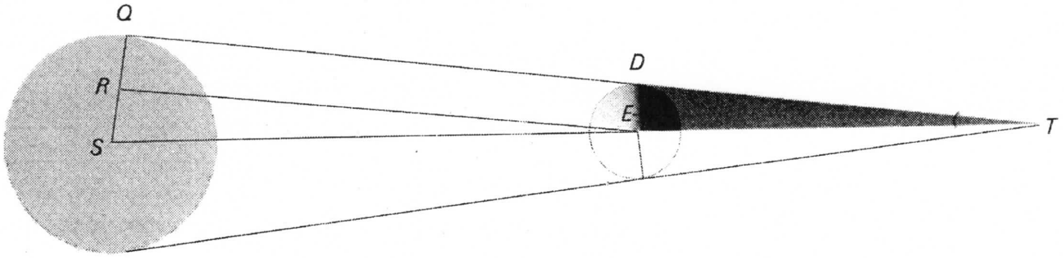


FIG. 9-12 Calculation of a shadow length.

(c) *The Lengths of Shadows**

The length of the shadow cast by a sphere in sunlight is easily computed; the principle is illustrated in Figure 9-12. The edge of the shadow *DT* is an extension of line *QD*, which is tangent to both the sun and the sphere. We construct line *RE*, parallel to *QD*, and note that the two triangles *SRE* and *EDT* are similar. From this fact, the equation

$$\frac{ET}{SE} = \frac{ED}{SR}$$

follows directly. Line *SE* is the distance between the centers of the sphere and sun. Line *ED* is the radius of the sphere, and line *SR* is the difference between the radii of the sun and the sphere. Line *ET*, of course, is the desired shadow length. Thus, we have

$$\text{shadow length} = \frac{\text{distance from sun} \times \text{radius of sphere}}{\text{radius of sun} - \text{radius of sphere}}$$

As an example, let us calculate the average length of the earth's shadow. The radii of the sun and earth are 432,000 and 3963 mi, respectively, and the mean distance of the sun is about 93 million mi. We have, therefore,

$$\text{length of earth's shadow} = \frac{93{,}000{,}000 \times 3963}{432{,}000 - 3963} = 860{,}000 \text{ mi.}$$

(d) Eclipse Seasons

For the moon to appear to cover the sun and thus to produce a solar eclipse, it must be in the same direction as the sun in the sky, that is, it must be at the *new* phase. For the moon to enter the earth's shadow and produce a lunar eclipse, it must be opposite the sun; that is, it must be at the *full* phase. Eclipses occur, therefore, only at new moon and at full moon. If the orbit of the moon about the earth lay exactly in the plane of the earth's orbit about the sun – in the ecliptic – an eclipse of the sun would occur at every new moon and a lunar eclipse at every full moon. However, because the moon's orbit is inclined at about 5° to the ecliptic, the new moon, in most cases, is not *exactly* in line with the sun, but is a little to the north or to the south of the sun in the sky. Similarly, the full moon usually passes a little south or north of the earth's shadow.

However, if full or new moon occurs when the moon is at or near one of the *nodes* of its orbit (where its orbit intercepts the ecliptic), an eclipse can occur. The line through the center of the earth that connects the nodes of the moon's orbit is called the *line of nodes*. If the direction of the sun lies along, or nearly along, the line of nodes, new or full moon occurs when the moon is near a node, and an eclipse results. The situation is illustrated in Figure 9-13. The orientation of the moon's orbit, and the line of nodes, *nn′*, remains relatively fixed during a revolution of the earth about the sun. There are, therefore, just two places in the earth's orbit, points *A* and *B*, where the sun's direction lies along the line of nodes. It is only during the

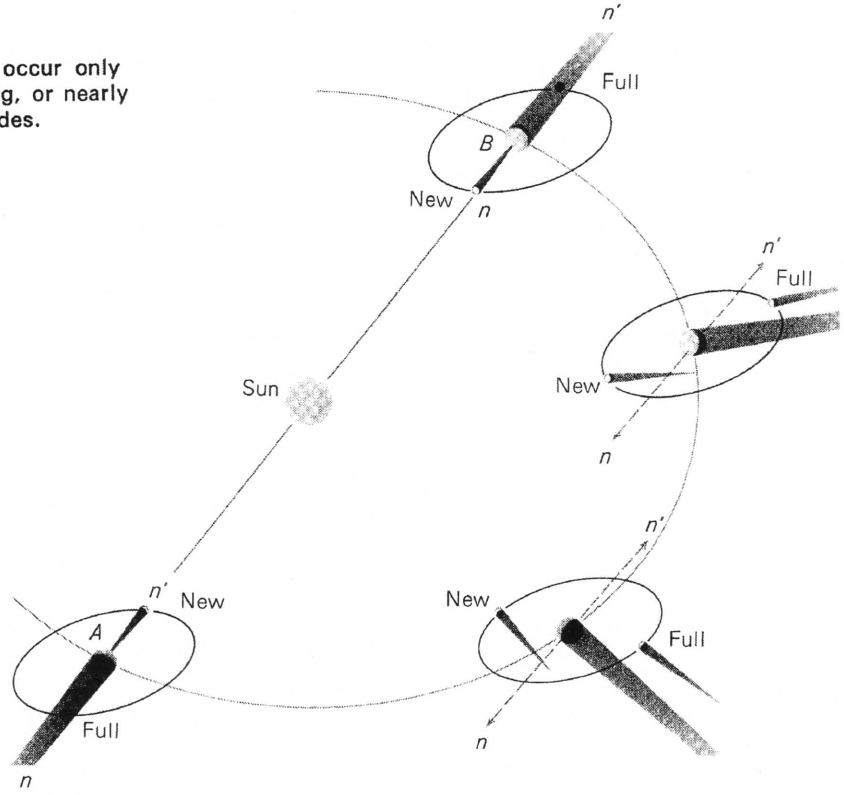


FIG. 9-13 Eclipses occur only when the sun is along, or nearly along, the line of nodes.

times in the year, roughly 6 months apart, when the earth-sun line is approximately along the line of nodes that eclipses can occur. These times are called *eclipse seasons.*

We have seen (Section 9.3) that because of perturbations of the moon's orbit, the line of nodes is gradually moving westward in the ecliptic, making one complete circuit in 18.6 years. Therefore, the eclipse seasons occur earlier each year by about 20 days. In 1970 the eclipse seasons are near February and August; in 1975 they are near November and May.

9.5 ECLIPSES OF THE SUN

One of the most surprising coincidences of nature is that the two most prominent astronomical objects, the sun and the moon, have so nearly the same apparent size in the sky. Although the sun is about 400 times as large in diameter as the moon, it is also about 400 times farther away, so both the sun and moon subtend about the same angle – about ½°.

The apparent or angular sizes of both sun and moon vary slightly from time to time, as their respective distances from the earth vary. The average angular diameter of the sun (as seen from the center of the earth) is 31′ 59″, and the average angular diameter of the moon is slightly less, 31′ 5″. However, the sun's apparent size can vary from the mean by about 1.7 percent and the moon's by 7 percent. The maximum apparent size of the moon is 33′ 16″, larger than the sun's apparent size, even at its largest. Therefore, if an eclipse of the sun occurs when the moon is somewhat nearer than its average distance, the moon can completely hide the sun, producing a *total solar eclipse.* An equivalent way of stating it is to say that a total eclipse of the sun occurs whenever the umbra of the moon's shadow reaches the surface of the earth.

(a) Geometry of a Total Solar Eclipse

The geometry of a total solar eclipse is illustrated in Figure 9-14. The earth must be at a position in its orbit such that the direction of the sun is nearly along the line of nodes of the moon's orbit. Furthermore, the moon must be at a distance from the surface of the earth that is less than the length of the umbra of the moon's shadow. Then, at new moon, the moon's umbra intersects the ground at a point *X* on the earth's surface. Anyone on the earth at *X* (within the small area covered by the moon's umbra) will not see the sun and will witness a total eclipse. The moon's penumbra, on the other hand, covers a larger area of the earth's surface. Any person within the penumbra will see part but not all of the sun eclipsed by the moon – a partial solar eclipse. The regions of total and partial eclipse correspond to points *A* and *B* in Figures 9-9 and 9-10.

As the moon moves eastward in its orbit at about 2100 mi/hr, its shadow sweeps eastward across the earth at the same speed. The earth, however, is rotating eastward at the same time, so the speed of the shadow with respect to a particular place on earth is less than 2100 mi/hr. At the equator, where the rotation of the earth carries places eastward at about 1040 mi/hr, the shadow moves relative to the earth with a speed of about 1060 mi/hr. In higher latitudes the speed is greater. In any case, the tip of the truncated cone of the umbra of the moon's shadow sweeps along a thin band across the surface of the earth, and the total solar eclipse is observed successively along this band (refer to Figure 9-14). This path across the earth within which a total solar eclipse is visible (weather permitting) is called the *eclipse path*. Within a zone about 2000 mi on either side of the eclipse path, a partial solar eclipse is visible – the observer, inside this limit, being located in the penumbra of the shadow.

Because the moon's umbra just barely reaches the earth, the width of the eclipse path, within which a total eclipse can be seen, is very small. Under the most favorable conditions, the path is only 167 mi wide in regions near the earth's equator. At far northern or southern latitudes, because the moon's shadow falls obliquely on the ground, it can cover a path somewhat more than 167 mi wide.

It does not take long for the moon's umbra to sweep past a given point on earth. The duration of totality may be only a brief instant. It can never exceed about 7½ minutes.

(b) Appearance of a Total Solar Eclipse

A total solar eclipse is one of the most spectacular of natural phenomena. If a person is anywhere near the path of totality of a solar eclipse, it is well worth his while to move into the eclipse path so that he may witness this rare and impressive event.

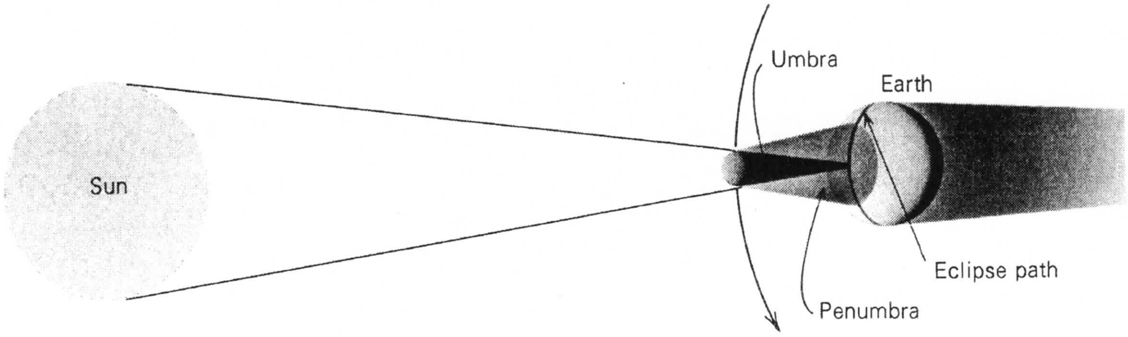


FIG. 9-14 Geometry of a total solar eclipse (not to scale).

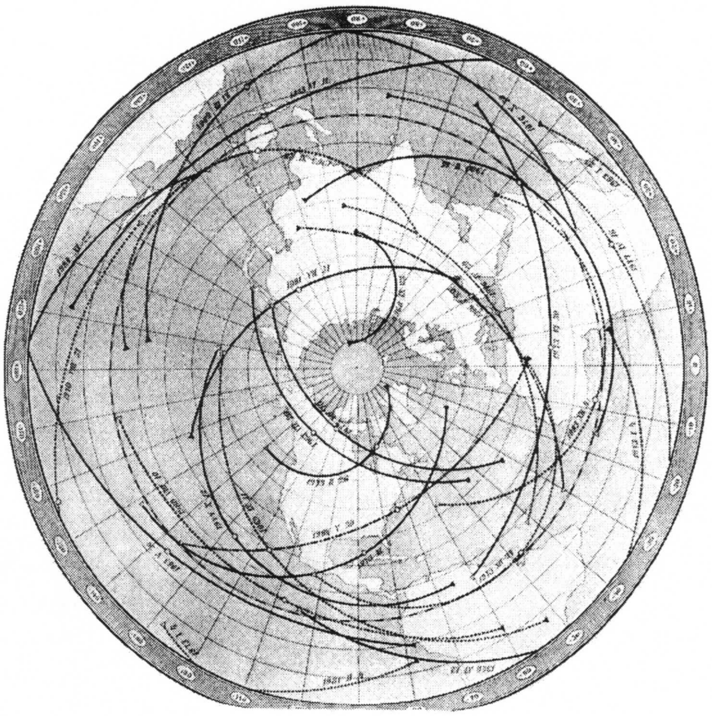


FIG. 9-15 Eclipse path of total solar eclipses occurring between 1963 and 1984. *(Yerkes Observatory.)*

The very beginning of a solar eclipse is the *first contact,* when the moon just begins to silhouette itself against the edge of the sun's disk. The *partial phase* of the eclipse is the period following first contact, during which more and more of the sun is covered by the moon. *Second contact* occurs from 1 to 2 hours after first contact, at the instant when the sun becomes completely hidden behind the moon. In the few minutes before second contact (the beginning of totality) the sky noticeably darkens; some flowers close up, and chickens may go to roost. Because the diminished light that reaches the earth must come solely from the edge of the sun's disk, and consequently from the higher layers in its atmosphere (see Chapter 24), the sky and landscape take on strange colors. In the last instant before totality, the only parts of the sun that are visible are those that shine through the lower valleys in the moon's irregular profile and line up along the periphery of the advancing edge of the moon – a phenomenon called *Baily's beads.* During totality, the sky is quite dark and the brighter stars are visible.

As Baily's beads disappear and the bright disk of the sun becomes entirely hidden behind the moon, the *corona* flashes into view. The corona is the sun's outer tenuous atmosphere, consisting of sparse gases that extend for millions of miles in all directions from the apparent surface of the sun. It is ordinarily not visible because the light of the corona is very feeble compared to that from the underlying layers of the sun that radiate most of the solar energy into space. Only when the brilliant glare from the sun's visible disk is blotted out by the moon during a total eclipse is the pearly white corona, the sun's outer extension, visible. Recently, however, it has become possible to photograph the inner, brighter, part of the corona with an instrument called a coronagraph, a telescope in which a black disk in the telescope's focal plane produces an artificial eclipse, enabling at least the brighter part of the corona to be studied at any time.

Also, during a total solar eclipse, the chromosphere can be observed – the layer of gases just above the sun's visible surface. Prominences, great jets of gas extending above the sun's surface, are sometimes viewed. These outer parts of the sun's atmosphere are discussed more completely in Chapter 24.

The total phase of the eclipse ends, as abruptly as it began, with *third contact,* when the moon begins to uncover the sun. Gradually the partial phases of the eclipse repeat themselves, in reverse order. At *last contact* the moon has completely uncovered the sun.

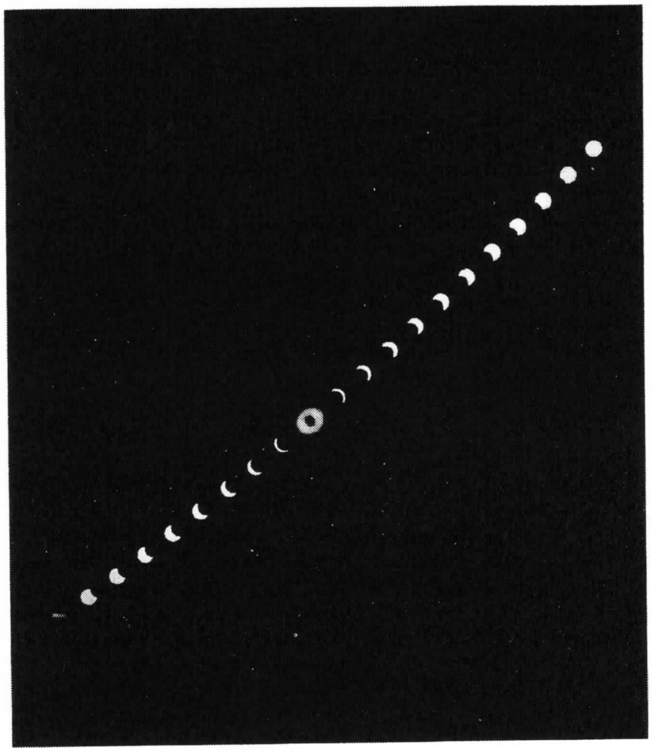
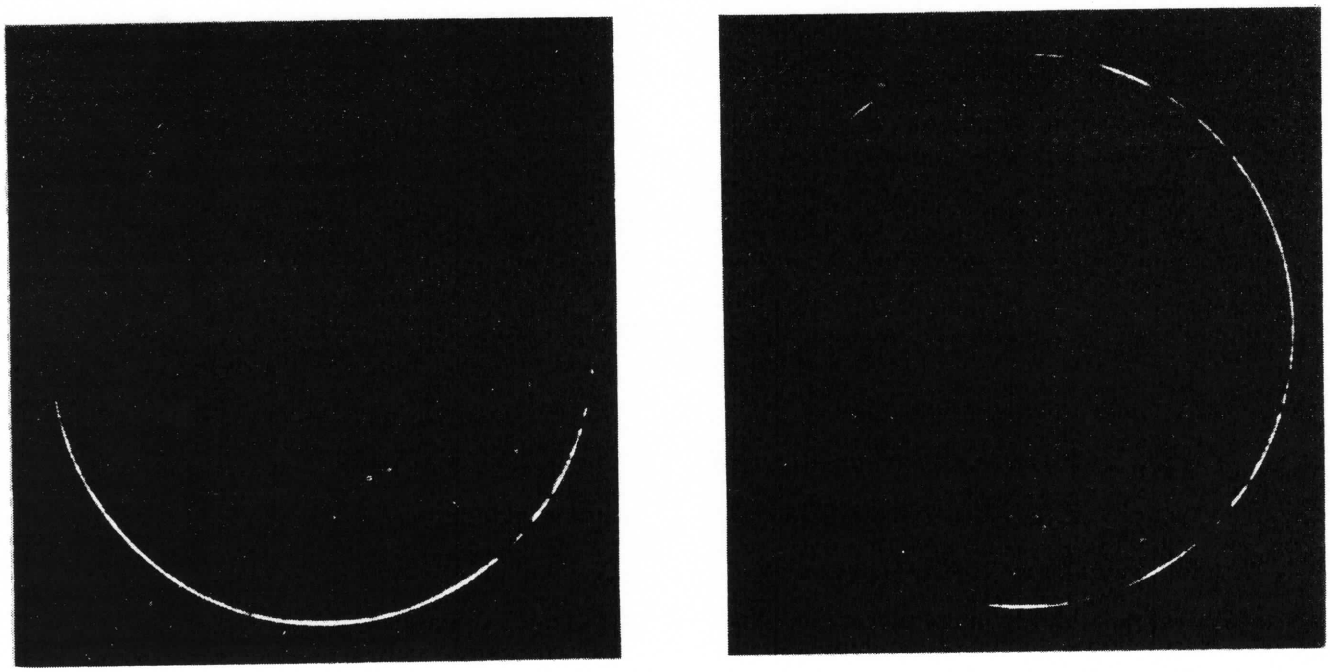


FIG. 9-16 Time-lapse photograph showing the moon passing in front of the sun during a total solar eclipse. *(American Museum of Natural History.)*

FIG. 9-17 Baily's beads, photographed by Van Biesbröck, May 20, 1947. *(Yerkes Observatory.)*



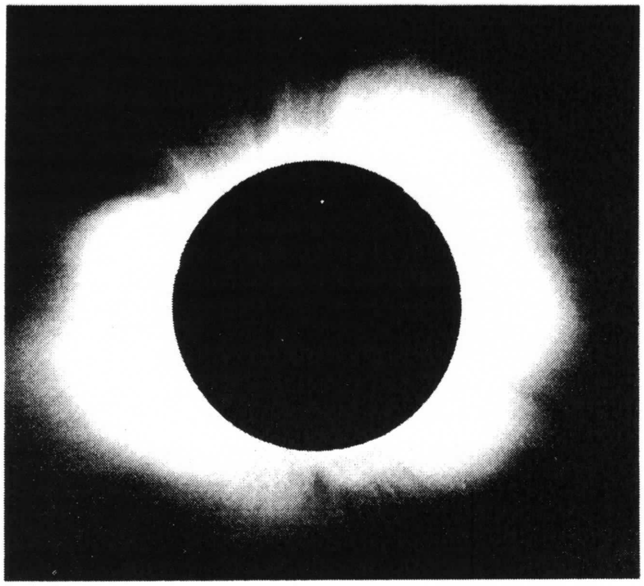


FIG. 9-18 The corona of the sun, photographed during the eclipse of June 8, 1918. *(Mount Wilson and Palomar Observatories.)*

According to ancient Chinese legend, an eclipse was an occasion when a giant dragon started to swallow the sun. People would therefore beat sticks and make noise to frighten the dragon away. Their noisemaking always did the job; sooner or later, the dragon would disgorge the sun, leaving it as whole as before.

(c) Value of Total Solar Eclipses

In addition to being beautiful to watch, total eclipses of the sun have considerable astronomical value. Many data are obtained during eclipses that are otherwise not accessible. For example, during an eclipse we can determine the exact relative positions of the sun and moon by timing the instants of the four contacts. We can take direct photographs and make spectrographic observations of the sun's outer atmosphere and prominences. We can measure the light and heat emitted by the corona. We can determine how meteorological conditions are affected by solar eclipses and can learn something about the light-scattering properties of the earth's atmosphere.

Historically, one of the most important of eclipse observations concerns the apparent positions of stars in the sky near the sun during totality. According to the general theory of relativity, light rays should be slightly deflected when passing near a massive body such as the sun. This means that those stars whose light rays pass very close to the disk of the sun in the sky should appear in a slightly displaced direction (Figure 9-19). It is possible to observe stars whose directions are close to that of the sun only during total solar eclipses. At other times, the bright glare of sunlight hides the stars, even from telescopic observation. The procedure for applying this test of general relativity is to photograph the star field near the sun during total eclipse and then to compare the measured positions of the star images with those observed when the same stars are photographed, by the same instruments, at other times of the year, when the sun is elsewhere in the celestial sphere. Unfortunately, the expected deflections of starlight passing near the sun are small and are consequently very difficult to measure. Positive results of this test of general relativity have, however, been reported. Most experts feel that it has been established that this gravitational deflection of starlight exists, although there is still some argument over the amount. At least the observations have been shown to be not inconsistent with general relativity.

(d) Annular Eclipses of the Sun

More than half the time the moon does not appear large enough in the sky to cover the sun completely, which means that the umbra of its shadow does not reach all the way to the surface of the earth. The geometry of the situation is illustrated in Figure 9-20. When the moon's shadow cone does not reach the earth, a total eclipse is not possible. However, if the boundaries of the umbra of the

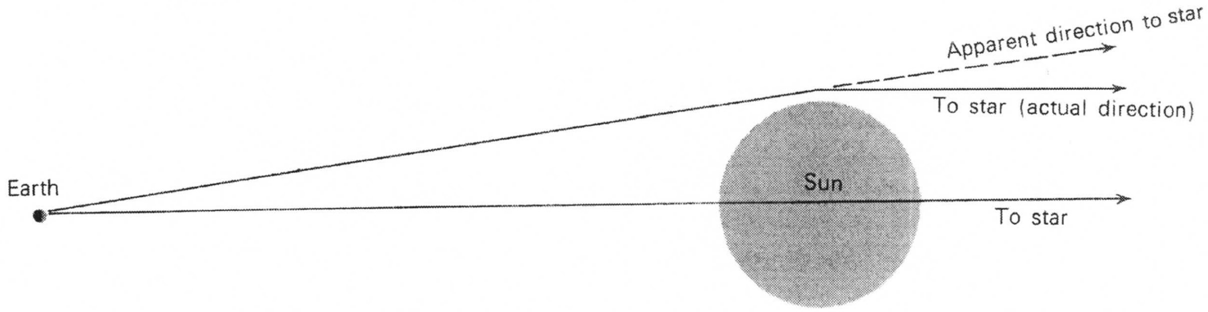


FIG. 9-19 Gravitational deflection of starlight passing near the sun — a test for general relativity.

shadow are extended until they intersect the earth's surface, they define a region on the ground within which the moon can be seen completely silhouetted against the sun's disk, with a ring of sunlight showing around the moon. This kind of eclipse is called an *annular eclipse,* from the Latin word *annulus,* meaning "ring." The extension of the moon's umbra, within which an annular eclipse is visible, sweeps across the ground in a path much like the path of totality of a total eclipse. As is true for total eclipses, a partial eclipse is visible within a region of 2000 mi or more on either side of the annular eclipse path. In this case, the regions of annular and partial eclipse correspond to positions *D* and *C*, respectively, in Figures 9-9 and 9-10.

An annular eclipse begins and ends like a total eclipse. However, because the sun is never completely covered by the moon, the corona is not visible, and although the sky may darken somewhat, it does not get dark enough for stars to be seen. An annular eclipse is not so spectacular, nor has it the scientific value of a total eclipse.

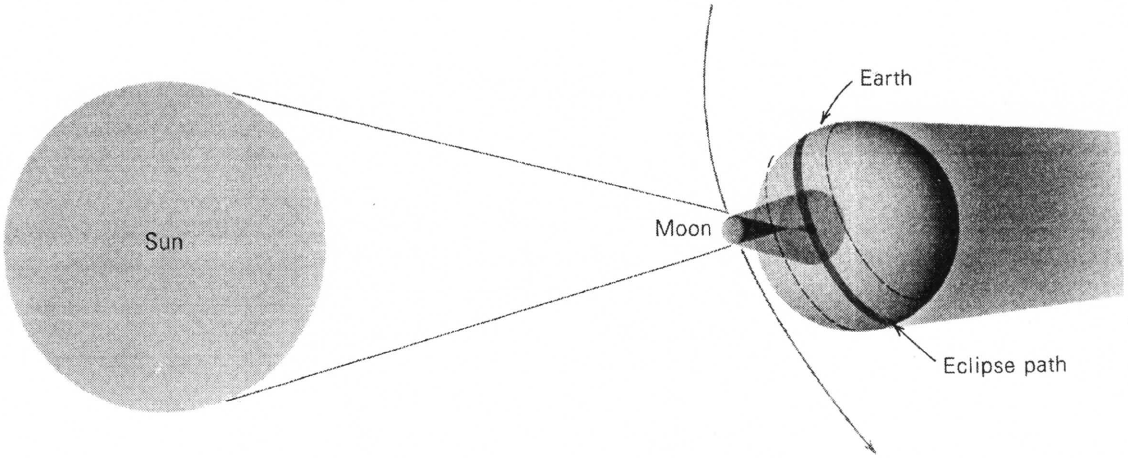


FIG. 9-20 Geometry of an annular eclipse (not to scale).

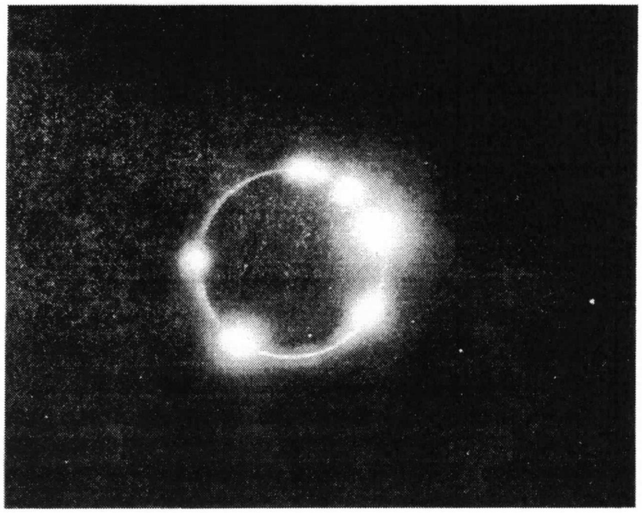


FIG. 9-21 An annular eclipse of the sun. Bright spots are sunlight streaming through lunar valleys. *(Lick Observatory.)*

Sometimes, because of the earth's finite size and sphericity, the umbra of the moon's shadow may be long enough to reach the surface of the earth only near the time when the moon is most nearly in a line between the centers of the earth and sun. Then it falls short of the earth's surface over the beginning and end of the eclipse path (see Figure 9-22). These mixed annular and total eclipses are relatively rare.

(e) Partial Eclipses of the Sun

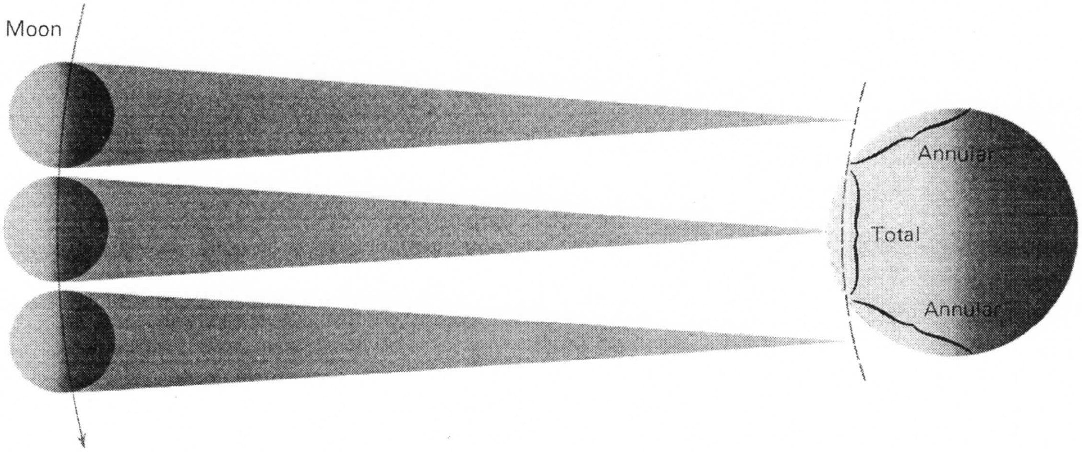
A *partial eclipse* of the sun is one in which only the penumbra of the moon's shadow strikes the earth. During such an eclipse, the moon's umbra passes north or south of the earth, and from nowhere can the sun appear to be covered completely by the moon. Also, a total or annular eclipse appears partial from regions outside the eclipse path but within the zone of the earth that is intercepted by the moon's penumbra. Few people, therefore, have seen total or annular solar eclipses, whereas most have had the opportunity to see the sun partially eclipsed. The moon seems to "skim" across the northern or southern part of the sun. How much of the sun can appear covered depends, of course, on how close the observer is to the path of totality or annularity. Partial eclipses are interesting but not spectacular. The progress of the eclipse can be observed conveniently through heavily smoked glass or densely exposed photographic film. Only if the observer is within a few hundred miles of the eclipse path will he see the sky darken appreciably.

No!

9.6 ECLIPSES OF THE MOON

A lunar eclipse occurs when the moon, at the full phase, enters the shadow of the earth. There are

FIG. 9-22 Mixed annular and total eclipses.



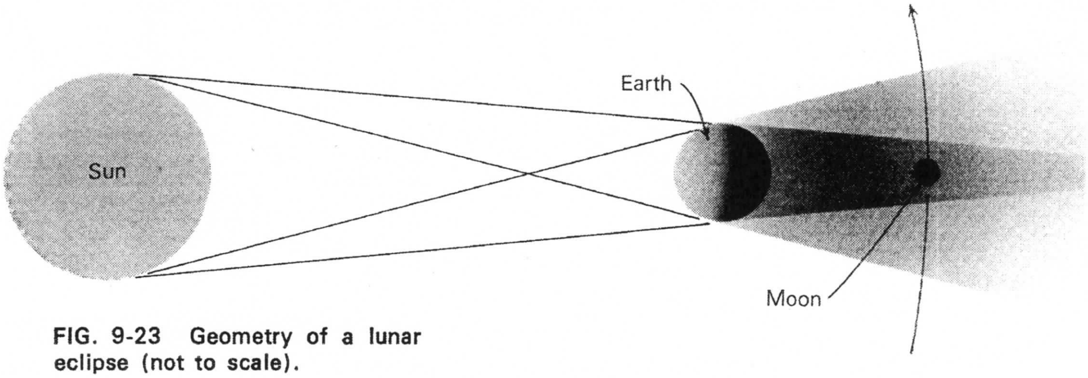


FIG. 9-23 Geometry of a lunar eclipse (not to scale).

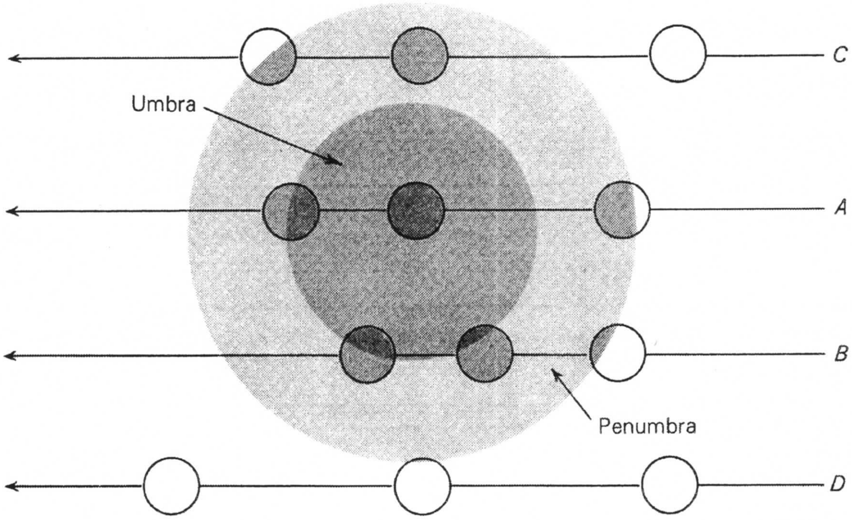


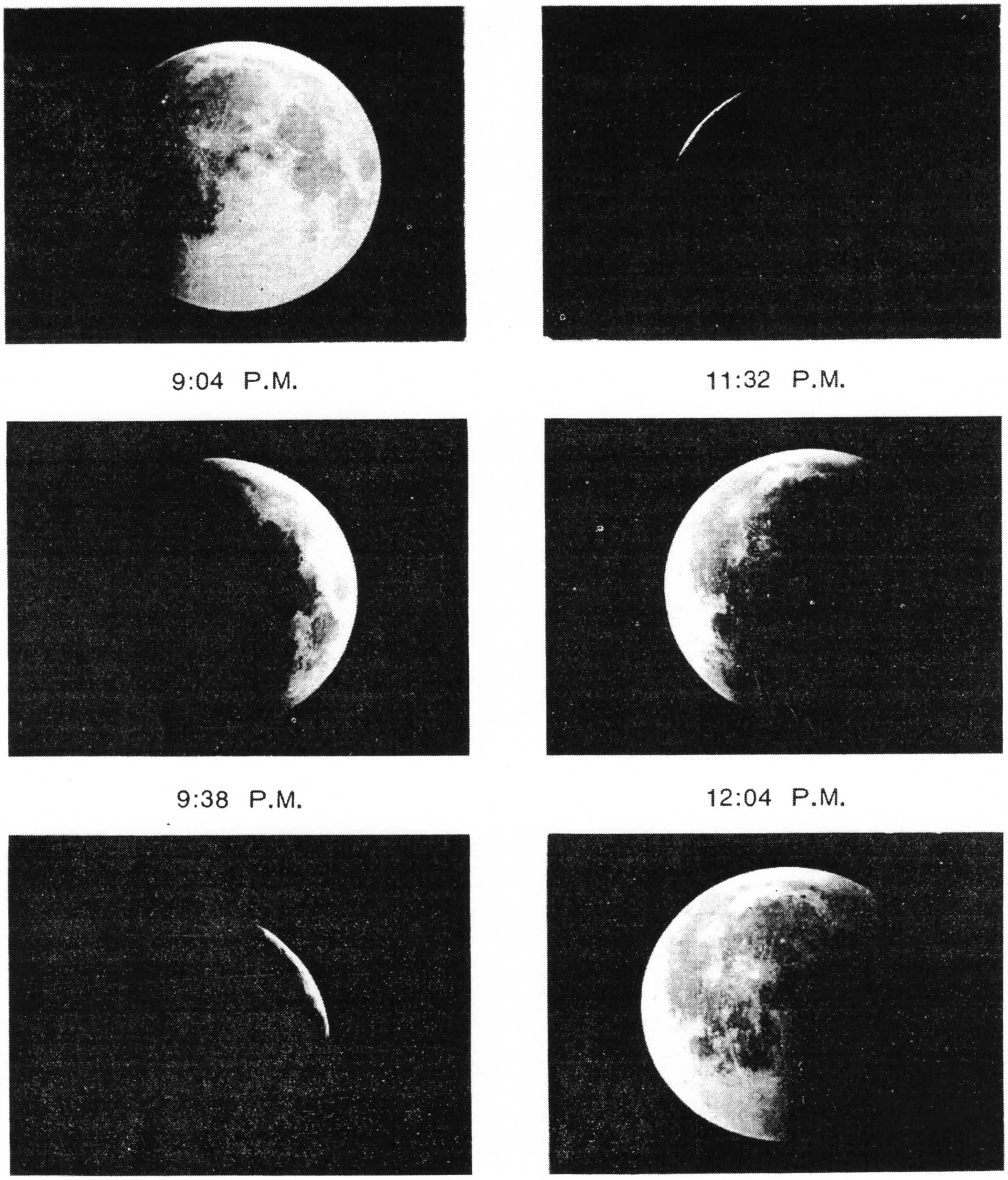
FIG. 9-24 Different kinds of lunar eclipses.

three kinds of lunar eclipses: *total, partial,* and *penumbral.*

(a) Geometry of a Lunar Eclipse

The geometry of lunar eclipses is shown in Figures 9-23 and 9-24. Unlike a solar eclipse, which is visible only in certain local areas on the earth, a lunar eclipse is visible to everyone who can see the moon. Weather permitting, a lunar eclipse can be seen from the entire night side of the earth, including those sections of the earth that are carried into the earth's umbra while the eclipse is in progress. A lunar eclipse, therefore, is observed far more frequently from a given place on earth than is a solar eclipse.

Figure 9-24 shows the cross section of the earth's shadow at the moon's distance. Since the cross-sectional diameter of a cone is proportional to the distance from the apex of the cone, the cross section of the earth's shadow cone at the moon's distance is in the same proportion to the size of the earth as the moon's distance from the end of the shadow is to the total length of the shadow. The umbra is thus found to be 5700 mi in diameter at the moon's distance. The value varies slightly from one eclipse to another because the earth-moon distance varies, and because the diameter of the umbra at the place where the moon enters the shadow depends on the moon's and the sun's distance at the time of the eclipse. The penumbra of the earth's shadow averages about 10,000 mi across at the moon's distance.



9:04 P.M.

11:32 P.M.

9:38 P.M.

12:04 P.M.

10:01 P.M.

12:23 P.M.

FIG. 9-25 Sequence of photographs of the total lunar eclipse of November 17/18, 1956. *(Photographed by Paul Roques, Griffith Observatory.)*

In Figure 9-24 are shown four of the many possible paths of the moon through the earth's shadow. A total lunar eclipse occurs when the moon passes completely into the umbra (path *A*). A *partial eclipse* occurs if only part of the moon skims through the umbra (path *B*), and a *penumbral eclipse* occurs if the moon passes through the penumbra, or partially through the penumbra, but does not come into contact with the umbra (paths *C* and *D*).

(b) Appearance of Lunar Eclipses

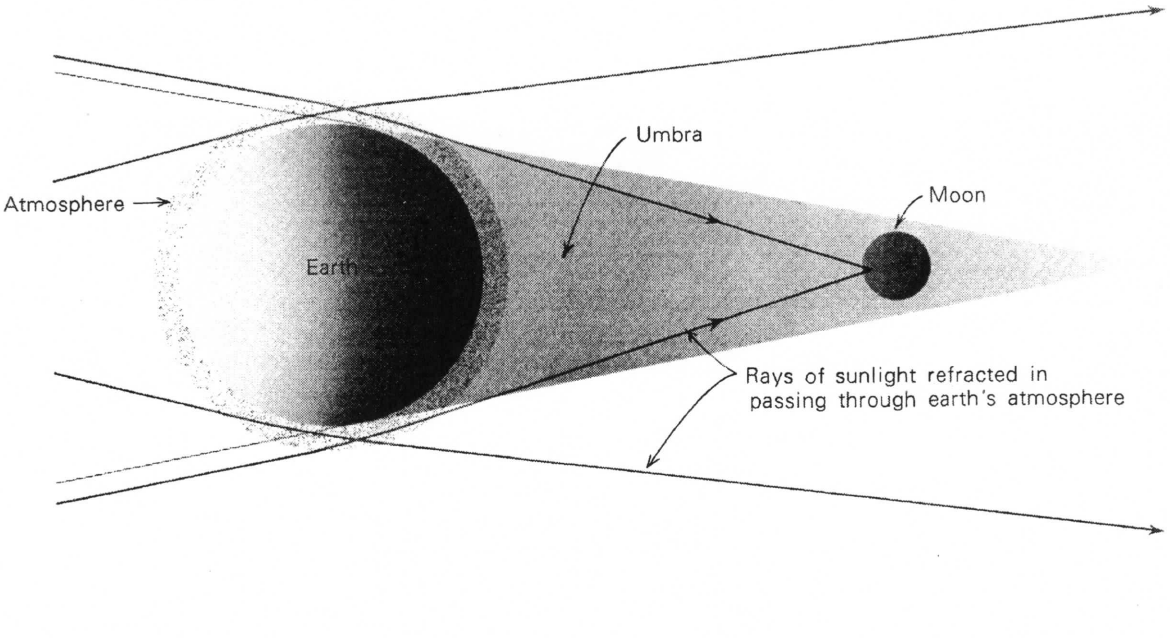
Penumbral eclipses usually go unnoticed even by astronomers. Only within about 700 mi of the umbra is the penumbra dark enough to produce a noticeable darkening on the moon. However, the diminished illumination on the moon's surface can be detected photometrically.

Every total or partial lunar eclipse must begin with a penumbral phase. About 20 minutes or so before the moon reaches the shadow cone of the earth, the side nearest the umbra begins to darken somewhat. At the moment called *first contact*, the limb of the moon (the "edge" of its apparent disk in the sky) begins to dip into the umbra of the earth. As the moon moves farther and farther into the umbra, the curved shape of the earth's shadow upon it is very apparent. In fact, Aristotle listed the round shape of the earth's shadow as one of the earliest proofs of the fact that the earth is spherical (Section 2.2d).

If the eclipse is a partial one, the moon never gets completely into the umbra of the earth's shadow but passes on by, part of it remaining in the penumbra, where it still receives some sunlight. At *last contact* the moon emerges from the umbra.

On the other hand, if the eclipse is a total one, at the instant of *second contact* the moon is completely inside the umbra, and the total phase of the eclipse begins. Even when totally eclipsed, the moon is still faintly visible, usually appearing a dull coppery red. Kepler explained this phenomenon in his treatise *Epitome*. The illumination on the eclipsed moon is sunlight that has passed through the earth's atmosphere and been refracted by the air into the earth's shadow (Figure 9-26).

FIG. 9-26 Illumination of the moon during a total eclipse by sunlight refracted by the earth's atmosphere into the earth's shadow.



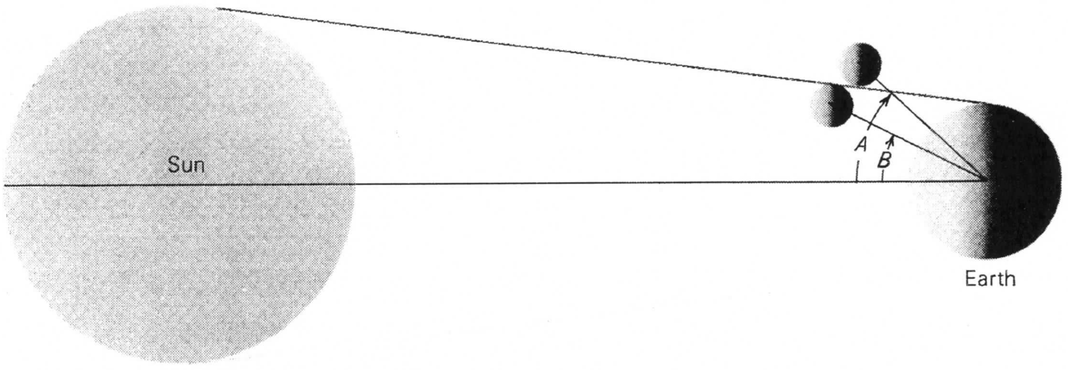


FIG. 9-27 Condition for a solar eclipse.

The eclipse will be darkest if the center of the lunar disk passes near the center of the umbra. The darkness of the lunar eclipse depends also on weather conditions around the terminator of the earth. It is only here, on the line between day and night on the earth, that sunlight passing through the atmosphere can be refracted into the shadow. Heavy cloudiness in that critical region will allow less light to pass through. The light striking the eclipsed moon inside the umbra is reddish because red light of longer wavelengths penetrates through the long path of the earth's atmosphere most easily.

Totality ends at *third contact,* when the moon begins to leave the umbra. It passes through its partial phases to *last contact,* and finally emerges completely from the penumbra. The total duration of the eclipse depends on how closely the moon's path approaches the axis of the shadow during the eclipse. The moon's velocity with respect to the shadow is about 2100 mi/hr; if it passes through the center of the shadow, therefore, about 6 hours will elapse from the time the moon starts to enter the penumbra until it finally leaves it. The penumbral phases at the beginning and end of the eclipse last about 1 hour each, and each partial phase consumes at least 1 hour. The total phase itself can last as long as 1 hour 40 minutes if the eclipse is central, or, of course, a shorter time if the moon does not pass through the center of the umbra.

9.7 ECLIPTIC LIMITS

We have seen that solar eclipses occur only when the moon is new and lunar eclipses when the moon is full. Furthermore, eclipses occur only when the new or full moon is near a node. If the sun, earth, and moon were geometrical points, the new or full moon would have to occur *exactly* at a node if there were to be an eclipse. Because of the finite sizes of the three bodies, however, part of the sun may appear covered by part of the moon for observers at certain places on earth even though at new moon the moon is not located *exactly* at the node. Similarly, in a lunar opposition, it is possible that, because of the large size of the earth's shadow, the full moon can pass partially through it or even pass completely into it, even if the full moon is not exactly at the node. Thus, the requirement that the moon be at a node can be relaxed slightly; there is some leeway within which eclipses can occur. The limits of this leeway are called *ecliptic limits.*

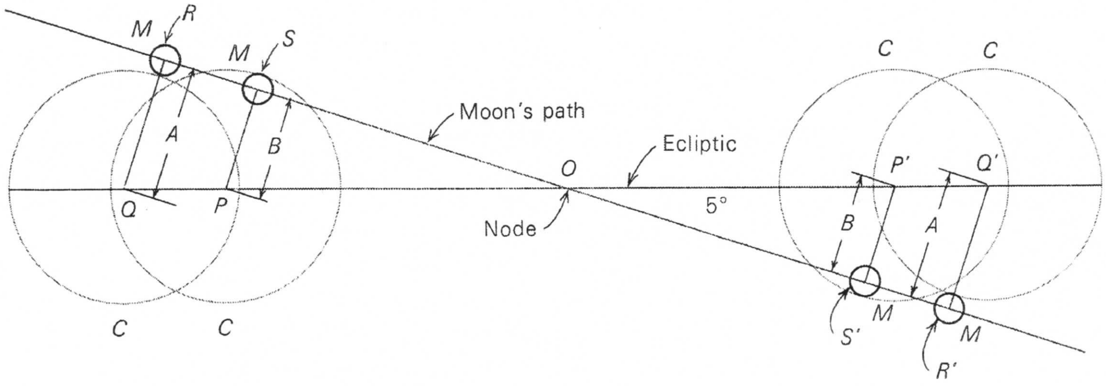
(a) *Solar Ecliptic Limits**

Note in Figure 9-27 that a partial eclipse of the sun is just barely visible somewhere on earth if the new moon encroaches on the conical surface enveloping the earth and sun. Under these conditions, it is a simple problem in geometry to find the angle *A* at the center of the earth between the centers of the sun and moon. Angle *A* turns out to be about 1½°. For an eclipse to be seen as *central,* that is, annular or total, the new moon must be completely inside the conical surface, in which case the angular separation, at the earth's center, between the centers of the moon and sun must be less than or equal to the angle *B; B* is about 1°.

In Figure 9-28 are shown the ecliptic and the moon's apparent path in the celestial sphere in the region of one of the nodes (the ascending node), which is indicated by *O.* The moon, *M,* is shown in four places in its path. The large circles, marked *C,* represent cross sections at the moon's distance of the cone tangent to the sun and earth. The centers of these circles, *Q, P, P',* and *Q',* are the positions of the sun's center at four different places along the ecliptic. Note that if new moon occurs at *R,* with the sun at *Q,* the centers of the sun and moon are separated by just the critical angle *A* for which some kind of eclipse is visible at some place on earth. Similarly, an eclipse can just barely occur if the sun and moon are at *Q'* and *R'.* If new moon occurs when the sun is outside the part of the ecliptic lying between *Q* and *Q',* an eclipse cannot occur. If the new moon is at *S* or *S',* and the sun at *P* or *P',* respectively, the two bodies are separated by the critical angle *B* required for a central eclipse. Central eclipses cannot take place if the new moon occurs when the sun is not within the region on the ecliptic between *P* and *P'.* The angular distance *OQ* or *OQ'* is called the *ecliptic limit* for a solar eclipse. The sun must be within this distance of the node at new moon to be eclipsed. The angular distance *OP* or *OP'* is called the *central ecliptic limit;* the sun must be within the central ecliptic limit of the node at new moon for an annular or total eclipse to occur. The angle of intersection between the moon's path and the ecliptic being known (about 5°), the actual values of the ecliptic limit and central ecliptic limit can be found easily by trigonometric calculation or geometrical construction. They are about 17 and 10 degrees, respectively.

The ecliptic limits can vary considerably, for they depend on the exact angular sizes of the sun and moon in the sky (and hence on their exact distances from the earth) and on the exact inclination angle of the moon's orbit to the ecliptic; all these are variable quantities. If we consider the extreme range of their variability, however, we can find the largest and smallest possible values for the ecliptic limits. These are called the *major* and *minor ecliptic limits,* respectively. The *major* and *minor solar ecliptic limits* are 18°31' and 15°21'. The *major* and *minor central ecliptic limits* are 11°50' and 9°55'. At the time of new moon, if the sun is farther from one of the moon's nodes than the major ecliptic limit, no eclipse can occur. If it is between the major and minor limits, an eclipse may occur; it is contingent upon the relative distances of the earth, sun, and moon at that instant, and upon the exact inclination of the moon's orbit. If the sun is within the minor ecliptic limit of a node at new moon, an eclipse must occur.

FIG. 9-28 Solar ecliptic limits.



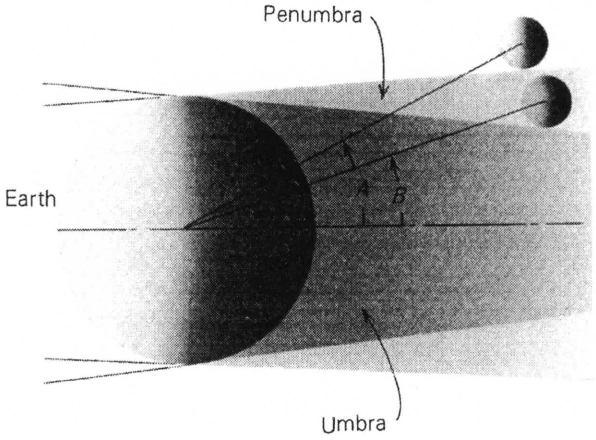


FIG. 9-29 Condition for a lunar eclipse.

(b) *Lunar Ecliptic Limits**

The *axis,* or center line, of the umbra of the earth's shadow is directed toward a point on the ecliptic exactly opposite the sun in the sky. If the moon is full while it is on or near the ecliptic (that is, near one of its nodes), it can enter the earth's shadow. In Figure 9-29 are shown the angles *A* and *B,* at the center of the earth, between the direction to the center of the moon and the axis of the umbra at which the moon can encroach, respectively, on the penumbra and umbra.

The penumbra shades gradually from the darkness of the umbra to full light outside; consequently, there is no observable phenomenon at the penumbra's outer boundary. The critical angle, *A,* for a penumbral eclipse, although well defined geometrically, has little real significance. Usually, we speak only of the limits of an umbral lunar eclipse (total or partial), for which the angle *B* applies. Angle *B* is a little less than 1°.

The *lunar ecliptic limit* is the maximum angle between the axis of the umbra and the direction of the node (or between the center of the sun and the opposite node) for which the moon can pass within an angular distance *B* of the umbral axis. The geometry of the situation, very similar to that of the solar ecliptic limit, is illustrated in Figure 9-30. If full moon occurs exactly at the node, *O,* the lunar eclipse is central. There will be no umbral eclipse if the moon passes the earth's shadow when it is farther from the node than at *Q* or *Q′*. The angle *OQ* or *OQ′* is thus the ecliptic limit for an umbral lunar eclipse. The angle varies somewhat, as does the solar ecliptic limit, because of the variations in the distances of the sun and moon and of the inclination of the moon's orbit. The *major* and *minor lunar ecliptic limits* are 12°15′ and 9°30′, respectively.

(c) *Frequency of Eclipses**

The angle through which the sun moves along the ecliptic during a synodic month averages 29°6′, and varies from this value only slightly. Between two successive new moons, therefore, the sun moves less than twice the minor solar ecliptic limit of 30°42′ (2 × 15°21′). Consequently, it is impossible for the sun to pass through a node without at least one eclipse during that eclipse season. There must, then, be at least two solar eclipses during any one calendar year. If the sun is eclipsed within a few days after it reaches the western ecliptic limit, a second eclipse can follow at the next new moon, just before the end of the eclipse season. If the first season falls in January, five solar eclipses are possible in a single calendar year, for that same eclipse season will begin again in the following December. This happened in 1935. Twice the *central* solar ecliptic limit is narrower (at most 23°40′), and no more than one total or annular eclipse of the sun is possible during an eclipse season; the number during a calendar year varies from none to three.

Eclipses of the moon are about as frequent as those of the sun if penumbral eclipses are counted, the number per year ranging from two to five. Umbral eclipses, on the other hand, occur about as often as central solar eclipses; there are from none to three in any one year.

Penumbral eclipses are inconspicuous, so the total number of observable solar eclipses over a period of time outnumbers the observable lunar eclipses by nearly three to two. Lunar eclipses are more common at any one station, however, for they can be viewed from more than half of the globe, whereas solar eclipses, even partial ones, are visible only in limited areas. Total solar eclipses occur on an average about once every 1½ years, but they are visible only within narrow eclipse paths; their average frequency at any one place is about once every 360 years.

The maximum number of all kinds of eclipses (solar and lunar) in any one calendar year is seven.

9.8 RECURRENCE OF ECLIPSES*

Thousands of years ago the ancients noticed that similar eclipses occurred at regular intervals. This recurrence of eclipses made it possible for early astronomers to predict eclipses with fair accuracy.

(a) *Circumstances For Two Similar Eclipses**

In order that an eclipse may be followed, after a lapse of time, by another eclipse very similar to the first one, the following conditions must be met: (1) the moon must be at the same phase again (new for a solar, full for a lunar eclipse); (2) the moon, when at that phase, must be in the same place in its orbit with respect to the node; and (3) the sun and moon must have the same distances from the earth again. If, in addition, the two eclipses are to have similar eclipse paths on the earth, they must occur at about the same time of the year.

Let us consider requirements (1) and (2) first. For the moon to return to exactly the same phase again, an integral (or whole) number of synodic months must have elapsed. To return to the same place in its orbit again with respect to the node, the moon must have made an integral number of revolutions about its orbit with respect to the nodes; that is, there must be an integral number of *nodical* or *draconic months.* (The term "draconic" is derived from the ancient Chinese superstition that eclipses were caused by dragons swallowing the sun.)

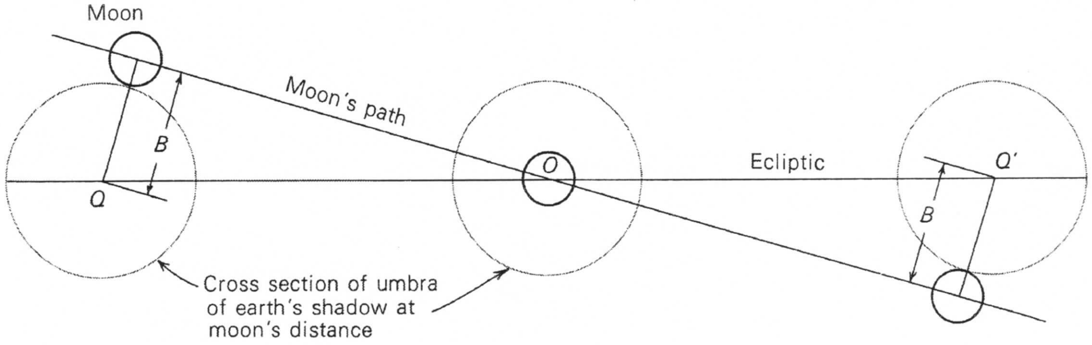
Suppose the synodic and nodical months were exactly 30 and 27 days long, respectively. Then eclipses nearly identical to each other would occur every 270 days, because both 9 synodic months and 10 nodical months would add up to 270 days. However, the synodic month is actually 29.5306 days and the nodical month is 27.2122 days, and there exists no integral least common multiple of these two numbers.

There are, on the other hand, some periods of time that are very nearly integral multiples of both the synodic and the nodical months. For example, the interval of 47 synodic months is almost the same as the interval of 51 nodical months:

47 synodic months = 1387.938 days;
51 nodical months = 1387.822 days.

As an illustration of the significance of this coincidence, suppose there is a solar eclipse. On the forty-seventh new moon following that eclipse, there will be another eclipse in which the moon is only about one tenth of a day's journey beyond the original place in its orbit relative to the node. Since the moon moves about 13° with respect to the node during 1 full day, this second eclipse will occur at a time when the sun and the moon are situated only a little over 1° from where they were, relative to the node, at the time of the eclipse 47 months earlier. Suppose the first eclipse had occurred just inside the western ecliptic limit. The second eclipse will take place a little farther inside the limit; the eclipse path on the earth will lie at latitudes slightly different from those of the first path. After another 47 synodic months, a third eclipse will occur, again about 1° further in from the western end of the ecliptic limit, and its path on the earth will again be displaced only slightly from that of the second eclipse. There will be a series of similar eclipses, occurring at intervals of 47 months, or about 1388 days. The

FIG. 9-30 Lunar ecliptic limits.



series of eclipses will continue until the position of the sun and moon during eclipse, shifting a little over a degree eastward from one eclipse to the next, has progressed along the ecliptic from the western end of the limit to the eastern end thereof, through a range of about 35°. There will be about 30 eclipses in this series.

Any two successive eclipses of the series described in the previous paragraph will have similar geometry but they will not be identical; some will be total and others annular. For identical eclipses, the relative distances of the earth, sun, and moon must be the same in both cases. Because the eccentricity of the earth's orbit about the sun is rather small, we may, as a first approximation, regard the moon's changing distance from the earth as the factor that determines the type of eclipse. The moon's closest approach to the earth (perigee) occurs at intervals of the anomalistic month of 27.55455 days. The anomalistic month differs from the sidereal and nodical months because the major axis of the moon's elliptical orbit (the line of *apsides*) gradually changes orientation in the plane of the moon's orbit (Section 9.3). The successive eclipses in an eclipse series are of about the same type if the integral number of synodic months is not only nearly equal to an integral number of nodical months but also is nearly equal to an integral number of anomalistic months. In Table 9.1 the various intervals are given for four eclipse cycles.

TABLE 9.1 Intervals of Eclipses in Four Representative Cycles

	47 synodic months	= 1387.938 days
1	51 nodical months	= 1387.822
	50 anomalistic months	= 1377.7275
	223 synodic months	= 6585.321
2	242 nodical months	= 6585.357
	239 anomalistic months	= 6585.538
	3803 synodic months	= 112,304.83
3	4127 nodical months	= 112,304.75
	4076 anomalistic months	= 112,312.35
	4519 synodic months	= 133,448.73
4	4904 nodical months	= 133,448.63
	4843 anomalistic months	= 133,448.69

(b) *Eclipse Series; the Saros**

Any of the above cycles can be used to predict eclipses; it is simplest to use the synodic month as the unit of time. Note that the second and fourth cycles contain nearly integral numbers of nodical, anomalistic, and synodic months. In both of those cycles the successive eclipses are of about the same type. In the fourth cycle, however, the interval is so long that changes in the orbit of the moon become significant. Most useful for the prediction of eclipses, therefore, is the second cycle of 223 synodic months, just 10 or 11 days in excess of 18 years. This cycle is called the *saros.*

Nearly identical eclipses, both solar and lunar, recur at intervals of the saros. Successive eclipses, following each other at these 18-year intervals, are said to belong to the same series. Of course, many eclipses occur during any 18-year period; thus, many series of eclipses are in progress at once. Because the saros cycle contains so nearly an integral number of tropical years, at each successive eclipse in the cycle the earth's axis has almost the same orientation relative to the ecliptic plane (or to the earth-sun line); thus successive eclipses even occur at approximately the same latitudes and have similar paths over the earth's surface.

The first solar eclipse in any series belonging to the saros cycle is a partial eclipse that occurs when the sun and the moon are at an eastern ecliptic limit. If it is the descending node, the eclipse is just barely visible near the south pole of the earth. If the eclipse is at the eastern limit of the ascending node, it is just visible at the north pole. After 223 synodic months the next eclipse in the series occurs, with the moon just about 1/30 of a day's journey, or slightly under ½°, west of the same place relative to the node. Thus the eclipse occurs with the sun and moon about ½° farther inside the ecliptic limit. After about 70 successive eclipses separated by intervals of the saros (about 1200 years), the positions of the sun and moon at the times of the eclipses have shifted through the node to the western ecliptic limit, and the series ends. The first dozen eclipses in the series are only partial ones visible near one of the polar regions of the earth (north pole for the sun and moon at the ascending node, south pole for the sun and moon at the descending node). The last dozen eclipses are partial ones visible at the other polar region. The middle 45 eclipses of the series are total or annular, the eclipse paths gradually shifting in latitude from one pole to the opposite one during the series.

If the saros interval contained an exactly integral number of days, successive eclipses in a series would occur at the same time of day for each place on earth, and the eclipses would all be visible at nearly the same longitudes as well as at the same latitudes. As it is, however, the saros interval is 6585.32 days; the approximate ⅓-day remainder causes successive eclipses in the cycle to occur about one third of the way around the world from each other. However, every third eclipse in a series does follow a path lying in nearly the same terrestrial longitudes.

Series of lunar eclipses, each one similar to the preceding one, also occur at intervals of the saros. However, because the limits for umbral lunar eclipses are smaller than for solar eclipses, a series of lunar eclipses runs through only about 50 saroses, which requires about 870 years.

The saros interval contains a nearly integral number of anomalistic months, so any two successive eclipses in the same series will be of nearly the same type, that is, both total, both annular, and so on. Of course, the type of eclipse changes as the series progresses. Especially notable is the series of total solar eclipses to which belong the eclipses of 1937, 1955, and 1973, because the duration of totality of these eclipses is near the maximum possible.

Table 9.2 lists total solar eclipses of appreciable duration of totality that are visible from inhabited places on the earth during the interval 1950 to 2000 A.D. This interval contains nearly three saros cycles, and the similarity of successive eclipses in the same cycle is evident from inspection. All eclipses belonging to a given saros cycle are identified with the same letter in the table.

TABLE 9.2 Important Total Solar Eclipses in the Second Half of the Twentieth Century

SAROS CYCLE	DATE	DURATION OF TOTALITY (MIN)	WHERE VISIBLE
a	1952 Feb. 25	3.0	Africa, Asia
b	1954 June 30	2.5	North-Central U.S. (Great Lakes), Canada, Scandinavia, U.S.S.R., Central Asia
c	1955 June 20	7.2	Southeast Asia
e	1958 Oct. 12	5.2	Pacific, Chile, Argentina
f	1959 Oct. 2	3.0	Northern and Central Africa
g	1961 Feb. 15	2.6	Southern Europe
h	1962 Feb. 5	4.1	Indonesia
i	1963 July 20	1.7	Japan, Alaska, Canada, Maine
j	1965 May 30	5.3	Pacific Ocean, Peru
k	1966 Nov. 12	1.9	South America
a	1970 March 7	3.3	Mexico, Florida, parts of U.S. Atlantic coastline
b	1972 July 10	2.7	Alaska, Northern Canada
c	1973 June 30	7.2	Atlantic Ocean, Africa
d	1974 June 20	5.3	Australia
e	1976 Oct. 23	4.9	Africa, Indian Ocean, Australia
f	1977 Oct. 12	2.8	Northern South America
g	1979 Feb. 26	2.7	Northwest U.S., Canada
h	1980 Feb. 16	4.3	Central Africa, India
i	1981 July 31	2.2	Siberia
j	1983 June 11	5.4	Indonesia
k	1984 Nov. 22	2.1	Indonesia, South America
l	1987 March 29	0.3	Central Africa
a	1988 March 18	4.0	Philippines; Indonesia
b	1990 July 22	2.6	Finland, Arctic Regions
c	1991 July 11	7.1	Hawaii, Central America, Brazil
d	1992 June 30	5.4	South Atlantic
e	1994 Nov. 3	4.6	South America
f	1995 Oct. 24	2.4	South Asia
g	1997 March 9	2.8	Siberia, Arctic
h	1998 Feb. 26	4.4	Central America
i	1999 Aug. 11	2.6	Central Europe, Central Asia

9.9 PHENOMENA RELATED TO ECLIPSES

We have considered so far only eclipses that involve the sun, moon, and earth. However, there are phenomena with similar geometrical properties that involve other celestial bodies. Examples are *occultations* and *transits.*

(a) Occultations

The moon often passes between the earth and a star; the phenomenon is called an *occultation.* The stars are so remote that the shadow of the moon cast in the light of a star is extremely long and is, in fact, sensibly cylindrical. Because a star is virtually a point source, there is no penumbra. During an occultation, a star suddenly disappears as the eastern limb of the moon crosses the line between the star and observer. If the moon is at a phase between new and full, the eastern limb will not be illuminated and the star may appear to vanish mysteriously as the dark edge of the moon covers it. Because the moon moves through an angle about equal to its own diameter every hour, the longest time that an occultation can last is about 1 hour. It can have a much shorter duration if the occultation is not central. Geometrically, occultations are equivalent to total solar eclipses, except that they are total eclipses of stars other than the sun.

The sudden disappearance of a star behind the limb of the moon during an occultation is evidence that the moon has no appreciable atmosphere. If there were one, the star would fade gradually as the moon's limb approached it, because the starlight would traverse a long path of the lunar atmosphere. Occultations also demonstrate the extremely small angular sizes of the stars (owing to their great distances). If a star had an appreciable angular size it would require a perceptible time to disappear behind the moon, as is true during the partial phases of a total solar eclipse. Actually, the partial phases of occultations have been measured photoelectrically, but they are extremely brief, less than a few hundredths of a second. The angular sizes of stars cannot be observed directly in a telescope, but they can often be determined by various techniques (see Chapter 22). It has been possible to observe that stars of large computed angular size require longer to disappear behind the moon than those of small angular size. Observations of occultations of celestial radio sources have been useful in detecting the accurate positions and angular sizes of those objects.

In the past, occultations have been valuable for determining the exact position of the moon. Because the stars appear as points, their positions in the celestial sphere can be determined with high accuracy, whereas it is much more difficult to measure the exact position of the moon, which not only appears large, but reflects so much sunlight that the fainter stars around it are often invisible. On the other hand, if an occultation can be accurately timed, the exact direction of the moon as seen from the place on earth where the occultation is observed can be found. In modern times, occultations are less valuable for this purpose, because special cameras have been developed to photograph the moon against a background of comparison stars. Its position can thus be found with considerable precision, by measurement on the photographic plate.

Occultations of the brighter stars are listed in advance in various astronomical publications. The times and durations of the occultations and the places on earth from which they are visible are given. Also listed are the comparatively rare occultations of planets by the moon, and of stars by planets.

(b) Transits

A *transit* is a passage of an inferior planet (Mercury or Venus) across the front of the sun's disk at the time of inferior conjunction. Usually each of these planets appears to pass north or south of the sun, but it can pass in front of the sun if inferior conjunction occurs when the planet is near one of the nodes of its orbit – the points where its orbit crosses the ecliptic.

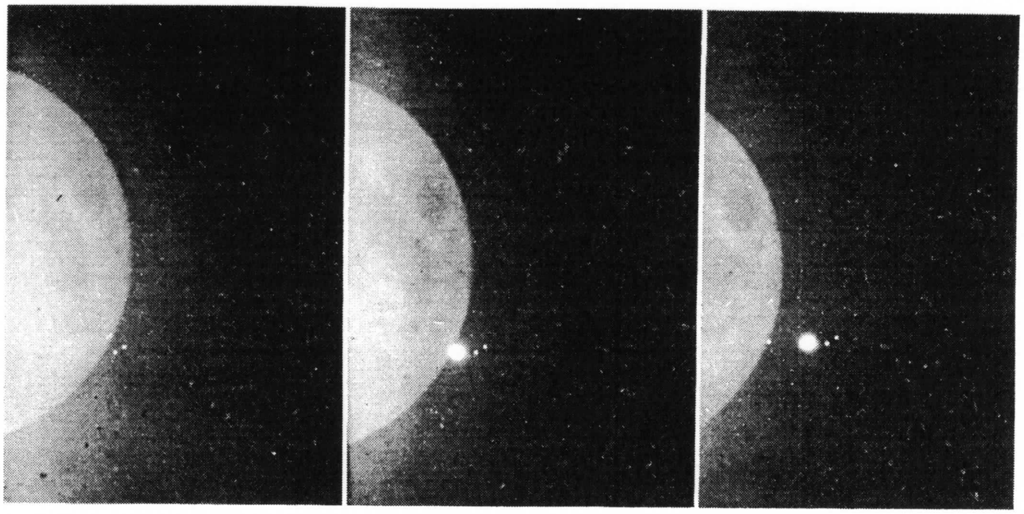


FIG. 9-31 Series of photographs showing *(left to right)* the emergence of Jupiter and three of its satellites after their occulation by the moon. *(Photographed by Paul Roques, Griffith Observatory.)*

The sun passes the nodes of Mercury's orbit in May and November; these are the seasons for transits of Mercury. However, transit "limits," analogous to ecliptic limits, are only a few degrees, and Mercury is seldom near enough to a node during the brief periods in May and November when transits of Mercury are possible. There are, on the average, about 13 transits of Mercury per century.

Transits of Venus are more rare than those of Mercury because the limits are narrower and because inferior conjunctions occur less frequently. Recurrences of transits occur just as recurrences of eclipses occur. Between two transits there must be an integral number of synodic periods of the planet, and there must be an integral number, or very nearly an integral number, of periods in which the planet has returned to the node. At present, transits of Venus occur in pairs separated by intervals of about 8 years. The last pair of Venusian transits were those of 1874 and 1882. The next pair will be on June 8, 2004, and June 6, 2012.

Transits are analogous to annular eclipses of the sun. The planets Venus and Mercury, seen from earth, are too small to cover the sun completely; their shadow cones fall far short of reaching the surface of the earth. The appearance of a transit is that of a black dot slowly crossing the disk of the sun from east to west. The silhouette of Mercury against the sun is too small to see without a telescope. That of Venus can be barely observed, without optical aid, if the sun is properly viewed by projection, or through dense filters to protect the eye.

(c) Eclipses on Other Planets

Eclipses would be visible from planets with satellites, other than the earth. For example, total solar eclipses would be common to observers living on the outside of the dense cloud layers that shroud the planet Jupiter. Several of Jupiter's 12 satellites regularly cast their shadows on the atmosphere of the planet. The dark spots on Jupiter where these shadows strike its atmosphere are easily visible through telescopes on earth. Only on the planets Mercury, Venus, and Pluto, which have no known satellites, are eclipses impossible.

(d) Stellar Eclipses

The most important kind of eclipses, to astronomers, are those of *eclipsing binary stars.* A binary system consists of two stars that revolve about each other. Such double stars are very common – many naked-eye stars are actually binary. When the plane of mutual revolution of such a double star happens to be so oriented that we see it edge on, each star periodically passes partially or entirely behind the other, thus being eclipsed. Then some or all of the light from the eclipsed star is prevented from reaching the earth, and the combined light received from the system is diminished. The most famous eclipsing binary star is the bright star *Algol* in the constellation of Perseus. Algol is really two stars revolving around each other, a bright one and a faint one. About every 2 days and 21 hours, the bright star passes partially behind the faint one and the apparent brightness of Algol drops down to less than half of normal – a change readily observable by the naked eye. A study of the light variation of such binary systems gives much information about the sizes and masses of their member stars. Eclipsing binary stars are described in Chapter 22.